UPSC Physics

Simulators for Practice

A PYQ repository

2023 Edition 1

Abhi Physics

MECHANICS

An UPSC CSE Physics Optional PYQ Repository

OCTOBER 3, 2023
ABHI PHYSICS
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Syllabus

(a) Mechanics of Particles:

- Laws of motion; conservation of energy and momentum, applications to rotating frames, centripetal and Coriolis accelerations;
- Motion under a central force; Conservation of angular momentum, Kepler's laws; Fields and potentials; Gravitational field and potential due to spherical bodies, Gauss and Poisson equations, gravitational self-energy; Two-body problem; Reduced mass; Rutherford scattering;
- Centre of mass a laboratory reference frame.

(b) Mechanics of Rigid Bodies:

- System of particles; Centre of mass, angular momentum, equations of motion;
- Conservation theorems for energy, momentum, and angular momentum; Elastic and inelastic collisions;
- Rigid body; Degrees of freedom, Euler's theorem, angular velocity, angular momentum, moments of inertia, theorems of parallel and perpendicular axes, equation of motion for rotation; Molecular rotations (as rigid bodies); Di and tri-atomic molecules; Processional motion; top, gyroscope.

(c) Mechanics of Continuous Media:

- Elasticity, Hooke's law and elastic constants of isotropic solids and their inter-relation;
- Streamline (Laminar) flow, viscosity, Poiseuille's equation, Bernoulli's equation, Stokes' law and applications.

(d) Special Relativity:

- Michelson-Morley experiment and its implications;
- Lorentz transformations-length contraction, time dilation, the addition of relativistic velocities, aberration, and Doppler effect, mass-energy relation, simple applications to a decay process;
- Four-dimensional momentum vector; Covariance of equations of physics.

Conservation Laws

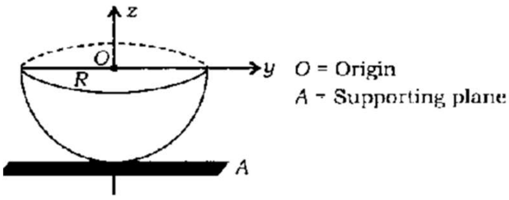
1. With an appropriate diagram, show that in the Rutherford scattering, the orbit of the particle is a hyperbola. Obtain an expression for impact parameter. TC 10 2011

2. Define a conservative field. Determine if the field given below is conservative in nature: TC 12 2012

$$\vec{E} = c\left[y^2\hat{\imath} + (2xy + z^2)\hat{\jmath} + 2yz\hat{k}\right]\mathrm{V/m}$$

where c is a constant.

3. Consider a uniform half-sphere of radius R and mass M. The half-sphere is supported by a frictionless horizontal plane as shown in the figure. The half-sphere lies in the region $z < 0$. TC 15 2012



Find the centre of mass of the half-sphere.

4. If the forces acting on a particle are conservative, show that the total energy of the particle which is the sum of the kinetic and potential energies is conserved. TC 20 2013
5. Prove that as a result of an elastic collision of two particles under non – relativistic regime with equal masses, the scattering angle will be 90°. Illustrate your answer with a vector diagram. TC 5 2013
6. Discuss the problem of scattering of charged particle by a coulomb field. Hence, obtain an expression for Rutherford scattering cross-section. What is the importance of the above expression? TC 25 2014
7. Write down precisely the conservation theorems for energy, linear momentum, and angular momentum of a particle with their mathematical forms. TC 10 2015
8. Draw a neat diagram to explain the scattering of an incident beam of particles by a centre of force. TC 10 2015

9. Show that the differential scattering cross-section can be expressed as TC 15 2015

$$\sigma(\theta) = \frac{\mathrm{s}}{\sin\ \theta}\left|\frac{\mathrm{ds}}{\mathrm{d}\theta}\right|,$$

where s is the impact parameter and θ is the scattering angle.

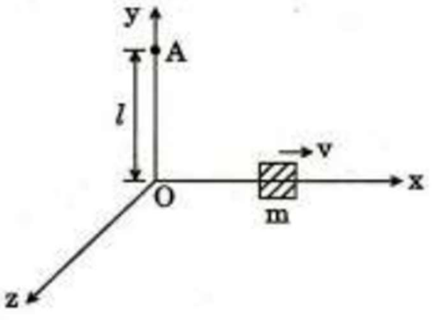
10. i. The distance between the centres of the carbon and oxygen atoms in the carbon monoxide (CO) gas molecule is 1.130×10^{-10} m. Locate the centre of mass of the molecule relative to the carbon atom. AN 10 2016

ii. Find the centre of mass of a homogeneous semicircular plate of radius a.

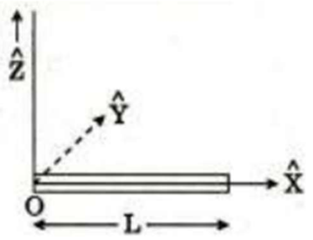
11. A diatomic molecule can be considered to be made up of two masses m_1 and m_2 separated by a fixed distance r. Derive a formula for the distance of centre of mass, C, from mass m_1. Also show that the moment of inertia about an axis through C and perpendicular to r is μr^2, where $\mu = \frac{m_1 m_2}{m_1 + m_2}$. TC 15 2017

12. A ball moving with a speed of 9 m/s strikes an identical stationary ball such that after the collision the direction of each ball makes an angle 30° with the original line of motion. Find the speed of the balls after the collision. Is the kinetic energy conserved in this collision? AN 15 2017

13. i. If a particle of mass m is in a central force field $f(r)\hat{r}$, then show that its path must be a plane curve, where $\hat{r}$ is a unit vector in the direction of position vector $\vec{r}$. TC 10 10 2018



ii. A block of mass m having negligible dimension is sliding freely in x-direction with velocity $\vec{v} = v\hat{\imath}$ as shown in the diagram. What is its angular momentum $\vec{\mathrm{L}}_{\mathrm{O}}$ about origin O and its angular momentum $\vec{\mathrm{L}}_{\mathrm{A}}$ about the point A on y-axis?

14.	A rod of length L has non-uniform linear mass density (mass per unit length) λ, which varies as $\lambda = \lambda_0 \left(\frac{\mathrm{S}}{\mathrm{L}}\right)$; where λ_0 is a constant and S is the distance from the end marked ' O ' (as shown in the figure). Find the centre of mass of the rod.  	TC	15	2018
15.	i. What is central force? Give two examples of the central force. ii. Show that the angular momentum $(\overrightarrow{\mathrm{L}})$ of the particle in a central force field is a constant of motion.	TC	10	2019
16.	Show that the cross-section for elastic scattering of a point particle from an infinitely massive sphere of radius R is $\frac{R^2}{4}$. What is the inference of this result?	TC	10	2019
17.	A rocket starts vertically upwards with speed v_0. Then define its speed v at a height h in terms of v_0, h, R (radius of Earth) and g (acceleration due to gravity on Earth's surface). Also calculate the maximum height attained by a rocket fired with a speed of 90% of the escape velocity.	TC	10	2020
18.	A particle moving in a central force field describes the path $r = \mathrm{k}e^{\alpha\theta}$, where k and α are constants. If the mass of the particle is m, find the law of force.	TC	10	2021
19.	i. Calculate the mass and momentum of a proton of rest mass 1.67×10^{-27} kg moving with a velocity of 0.8c, where c is the velocity of light. If it collides and sticks to a stationary nucleus of mass $5 \cdot 0 \times 10^{-26}$ kg, find the velocity of the resultant particle. ii. Calculate the mass of the particle whose kinetic energy is half of its total energy. Find the velocity with which the particle is travelling.	AN	8 7	2021
20.	Show that the mean kinetic and potential energies of non-dissipative simple harmonic vibrating systems are equal.	TC	10	2022
21.	Show that for very small velocity, the equation for kinetic energy, $K = \Delta mc^2$ becomes $K = \frac{1}{2} m_0 v^2$, where notations have their usual meanings.	TC	10	2022

Central Force Motion and Gravitation

22.	A particle P of mass m_1 collides with another particle Q of mass m_2 at rest. The particles P and Q travel at angles θ and ϕ, respectively, with respect to the initial direction of P. Derive the expression for the maximum value of θ.	TC	15	2022
23.	A planet revolves around the Sun in an elliptic orbit of eccentricity e. If T is the time period of the planet, find the time spent by the planet between the ends of the minor axis close to the Sun.	TC	10	2010
24.	A particle is moving in a central force field on an orbit given by $r = ke^{\alpha\theta}$, where k and α are positive constants, r is the radial distance and θ is the polar angle. (a) Find the force law for the central force field. (b) Find $\theta(t)$. (c) Find the total energy.	AN	20 20 20	2012
25.	A particle describes a circular orbit under the influence of an attractive central force directed towards a point on the circle. Show that the force varies as the inverse fifth power of distance.	TC	15	2013
26.	A charged particle is moving under the influence of a point nucleus. Show that the orbit of the particle is an ellipse. Find out the time period of the motion.	TC	15	2014
27.	The density inside a solid sphere of radius a is given by $\rho = \frac{\rho_0 a}{r}$, where ρ_0 is the density at the surface and r denotes the distance from the centre. Find the gravitational field due to this sphere at a distance 2 a from its centre.	AN	10	2014
28.	A body moving in an inverse square attractive field traverses on elliptical orbit with eccentricity e and period γ. Find the time taken by the body to traverse the half of the orbit that is nearer the centre of force. Explain briefly why a comet spends only 18% of its time on the half of its orbit that is nearer the sun.	TC	10	2016
29.	Express angular momentum in terms of kinetic, potential and total energy of a satellite of mass m in a circular orbit of radius r.	TC	10	2017
30.	Use Gauss's theorem to calculate the gravitational potential due to a solid sphere at a point outside the sphere. Calculate the amount of work required to send a	AN	15	2018

	body of mass m from the Earth's surface to a height $R/2$, where R is the radius of the Earth.			
31.	The radius of the Earth is 6.4×10^6 m, its mean density is 5.5×10^3 kg/m^3 and the universal gravitational constant is 6.66×10^{-1} Nm^2/kg^2. Calculate the gravitational potential on the surface of the Earth.	AN	10	2021

Rotating Frame

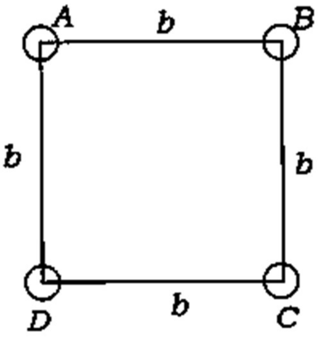
32.	With an appropriate diagram, show that in the Rutherford scattering, the orbit of the particle is a hyperbola. Obtain an expression for impact parameter.	TC	10	2011
33.	Prove that the time taken by the earth to travel over half of its orbit separated by the minor axis remote from the sun is two days more than half a year. Given, the period of the earth is 365 days and eccentricity of the orbit $= 1/60$.	TC	10	2011
34.	A rigid body is spinning with an angular velocity of $4\,\frac{\text{rad}}{\text{s}}$ about an axis parallel to the direction $(4\hat{j} - 3\hat{k})$ passing through the point A with $\overrightarrow{OA} = 2\hat{\imath} + 3\hat{j} - \hat{k}$, where O is the origin of the coordinate system. Find the magnitude and direction of the linear velocity of the body at point P with $\overrightarrow{OP} = 4\hat{\imath} - 2\hat{j} + \hat{k}$	AN	12	2012
35.	Suppose that an S'-frame is rotating with respect to a fixed frame having the same origin. Assume that the angular velocity $\vec{\omega}$ of the S$'$-frame is given by $\vec{\omega} = 2t\hat{\imath} - t^2\hat{j} + (2t+4)\hat{k}$ where t is time and the position vector $\vec{r}$ of a typical particle at time t as assumed in S'-frame is given by $\vec{r} = (t^2+1)\hat{\imath} - 6t\hat{j} + 4t^3\hat{k}.$ Calculate the Coriolis acceleration at $t = 1$ second.	AN	10	2013
36.	Calculate the horizontal component of the Coriolis force acting on a body of mass 0.1 kg moving northward with a horizontal velocity of 100 ms^{-1} at $30°$N latitude on the Earth.	AN	15	2013

37.	Derive the expression for Coriolis force and show that this force is perpendicular to the velocity and to the axis of rotation. What is the nature of this force?	TC	10	2016
38.	Consider two frames of reference S and S' having a common origin O. The frame S' is rotating with respect to the fixed frame S with a uniform $\vec{\omega} = 3a_x \mathrm{rads}^{-1}$. A projectile of unit mass at position vector $\vec{r} = 7a_x + 4a_y\ \mathrm{m}$ is moving with $\vec{v} = 14a_y\ \mathrm{m\ s}^{-1}$. Calculate in the rotating frame S' the following forces on the projectile: (i) Euler's force (ii) Coriolis force (iii) Centrifugal force	AN	15	2022

Rigid Body Dynamics

39.	A uniform solid sphere of radius R having moment of inertia I about its diameter is melted to form a uniform disc of thickness t and radius r. The moment of inertia of the disc about an axis passing through its edge and perpendicular to the plane is also equal to I. Show that the radius r of the disc is given by $r = \frac{2R}{\sqrt{15}}$.	TC	10	2010
40.	What are Eulerian angles? A body with rotational symmetry about an axis is rotating under gravity about a point on the axis without friction. What are the quantities remaining constant during the motion? Find them in terms of suitable Eulerian angles. Explain 'precession' and 'nutation' of such a body.	TC	25	2010
41.	Imagine that a rigid body is rotating about a fixed point with angular velocity $\vec{\omega}$. Assuming that the coordinate axes coincide with the principal axes, if T stands for kinetic energy and G for external torque acting on the body, show that $\frac{dT}{dt} = \vec{G} \cdot \vec{\omega}$	TC	10	2011
42.	Determine the number of degrees of freedom for a rigid body- (i) moving freely in space of three dimensions; (ii) having one point fixed; (iii) having two points fixed.	TC	30	2011

43.	Calculate the moment of inertia of a solid cone of mass M, height h, vertical half-angle α and radius of its base R, about an axis passing through its vertex and parallel to its base.	TC	12	2012
44.	Show that the kinetic energy and angular momentum of torque free motion of a rigid body is constant.	TC	10	2013
45.	If I' and I be the Moments of Inertia of a body about an axis passing through an arbitrary origin and about a parallel axis through the centre of mass respectively, show that $I' = MR^2 + I$, where $\vec{R}$ is the position vector of the centre of mass with respect to the arbitrary origin and M is the mass of the body.	TC	10	2014
46.	Consider a rigid body rotating about an axis passing through a fixed point in the body with an angular velocity $\vec{\omega}$. Determine the kinetic energy of such a rotating body in a coordinate system of principal axis. If the earth suddenly stops rotating, what will happen to the rotational kinetic energy? Comment in detail.	TC	25	2014
47.	A body turns a fixed point. Show that the angle between its angular velocity vector and its angular momentum vector about a fixed point is always acute.	TC	15	2014
48.	How does one obtain the angular velocity of the Earth about the North Pole with respect to a fixed star as $7.292 \times 10^{-5}\mathrm{sec}^{-1}$? Explain your method of calculating the above value.	TC	10	2015
49.	Show that the moment of inertia of a circular disc of mass M and radius R about an axis passing through its centre and perpendicular to its plane is $\frac{1}{2}\mathrm{MR}^2$.	TC	15	2015
50.	Four solid spheres A, B, C, and D each of mass m and radius a, are placed with their centres on the four corners of square of side b as shown in the figure below:	AN	20	2016



Calculate the moment of inertia of the system about one side of the square, Also, calculate the moment of inertia of the system about a diagonal of the square.

51. Define moment of inertia and explain its physical significance. Calculate the moment of inertia of an annular ring about an axis passing through its centre and perpendicular to its plane. TC 20 2017

52. TC AN 15 2019
    i. Find the moments of inertia of rigid diatomic molecule about different axes of symmetry through the centre of mass.
    ii. A proton is 1837 times heavier than an electron. Find the centre of mass of hydrogen atom.

53. Write down Euler’s dynamical equations of motion (no derivation) of a rigid body about a fixed point under the action of a torque. Show that the kinetic energy of the torque-free motion is constant. TC 10 2019

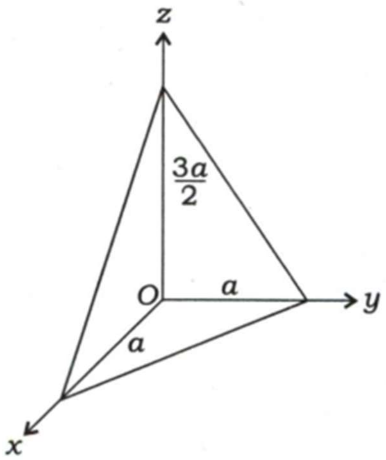
54. Where do you find the applications of gyroscope? AN 15 2019
A top of mass $0.200\ \mathrm{kg}$ is made up of a thin disc of radius $0.12\ \mathrm{m}$. It is pierced in the centre and a pin of negligible mass is mounted normal to its plane. The pivot under the disc is $0.03\ \mathrm{m}$ long. The top is made to spin with its axis making an angle $\theta = 20^\circ$ with the vertical and a precessional angular speed of $2\mathrm{rad/s}$. Calculate the angular speed with which it spins.

55. Determine the location of the centre of mass of a uniform solid hemisphere of radius R and mass M from the centre of its base. TC 10 2020

56. Obtain expressions for the moment of inertia of a solid cone about its TC 20 2020
    (i) Vertical axis and
    (ii) axis passing through the vertex and parallel to its base.

57. An electron is moving under the influence of a point nucleus of atomic number Z. Show that the orbit of the electron is an ellipse. TC 10 2021

58. A homogeneous right triangular pyramid with the base side a and height $\frac{3a}{2}$ is shown below. Obtain the moment of inertia tensor of the pyramid: AN 20 2021



Mechanics of Continuous media

59. When a sphere of radius r falls down a homogeneous viscous fluid of unlimited extent with the terminal velocity v, the retarding viscous force acting on the sphere depends on the coefficient of viscosity η, the radius r and its velocity v. Show how Stokes' law was arrived at connecting these quantities from the dimensional considerations. TC 10 2010

60. With an appropriate diagram, deduce the velocity profile for streamline flow of a liquid through a capillary of circular cross-section. Deduce also the fraction of liquid which flows through the section up to a distance a/2 from the axis, where a is the radius of the capillary. TC 20 2011

61. A sphere of radius R moves with velocity $\vec{\mathrm{u}}$ in an incompressible, non-viscous ideal fluid. Calculate the pressure distribution over the surface of the sphere. Do you think that a force is necessary to keep the sphere in uniform motion? TC 10 2014

62. Using Poiseuille's formula, show that the volume of a liquid of viscosity coefficient η passing per second through a series of two capillary tubes of lengths l_1 and l_2 having radii r_1 and r_2 is obtained as TC 15 2015

$$\mathrm{Q} = \frac{\pi \mathrm{p}}{8\eta} / \left[\frac{l_1}{\mathrm{r}_1^4} + \frac{l_2}{\mathrm{r}_2^4}\right],$$

where p is the effective pressure difference across the series.

63.	Define coefficients of viscosity and kinematic viscosity of a fluid. What are Poise and Stokes?	TC	10	2015
64.	Write down Poiseuille’s formula and mention its limitations in analysing the flow of a liquid through a capillary tube.	TC	10	2015
65.	Show that the Young's modulus Y, modulus of rigidity n and Poisson's ratio σ are related by the equation $Y = 2n(1+\sigma)$.	TC	10	2016
66.	A horizontal pipe of non-uniform bore has water flowing through it such that the velocity of flow is $40\ \mathrm{cm/s}$ at a point where the pressure is $2\ \mathrm{cm}$ of mercury column. What is the pressure at a point where the velocity of flow is $60\ \mathrm{cm/s}$? (Take, $g = 980\ \mathrm{cm/s^2}$ and density of water $= 1\ \mathrm{g/c.c.}$)	AN	10	2016
67.	State and explain Stokes' law. A drop of water of radius $0.01\ \mathrm{m}$ is falling through a medium whose density is $1.21\ \mathrm{kg/m^3}$ and $\eta = 1.8 \times 10^{-5}\ \mathrm{N-s/m^2}$. Find the terminal velocity of the drop of water.	AN	15	2017
68.	Two capillary tubes of lengths $2l$ and l with internal radii r and $2\mathrm{r}$ respectively are connected in series. Water flows through them in streamline. If the pressure difference across the first capillary is P, find the pressure difference across the second one.	TC	10	2018
69.	A water drop of radius $0.04\ \mathrm{mm}$ is falling through air. If the coefficient of viscosity for air is 1.8×10^{-4} poise, find its terminal velocity. If 100 such drops coalesce, what will be the new terminal velocity?	AN	10	2018
70.	i. How does Reynolds number help in the study of fluid motion? ii. In a horizontal pipeline of uniform area of cross-section, the pressure falls by $5\mathrm{Nm^{-2}}$ between two points separated by a distance of $1\ \mathrm{km}$. Calculate the change in kinetic energy per kg of oil flowing at these points. Density of oil $= 800\ \mathrm{kg\ m^{-3}}$.	AN	10	2019
71.	A rubber cord $1\ \mathrm{mm}$ in diameter and $1\ \mathrm{m}$ long is fixed at one end and a weight of $1\ \mathrm{kg}$ is attached to the other end. If the Young's modulus of rubber is 0.05×10^{11} dynes $\mathrm{cm^{-2}}$, then find the period of the vertical oscillations of the weight.	AN	10	2020

72. A shaft of diameter $8 \mathrm{~cm}$ and length $5 \mathrm{~m}$ is transmitting power of $8 \mathrm{~kW}$ at 300 revolutions per minute. If the coefficient of rigidity of the material of the shaft be 8×10^{11} dynes $/ \mathrm{cm}^2$, then calculate the relative shift between the ends of the shaft. AN 15 2020

73. A capillary tube having $1.0 \mathrm{~mm}$ diameter, $20 \mathrm{~cm}$ in length is fitted horizontally to a vessel in which alcohol is kept fully up to the neck. Density of alcohol is $8 \times 10^2 \mathrm{~kg} / \mathrm{m}^3$. The depth of the centre of the capillary tube below the surface of alcohol is $40 \mathrm{~cm}$. Find the amount of alcohol that will flow out of the capillary tube in 10 minutes. Coefficient of viscosity of alcohol is $0.0012 \mathrm{Ns} / \mathrm{m}^2$. AN 10 2021

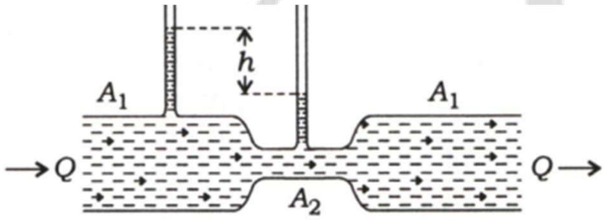
74. A light rod of length $100 \mathrm{~cm}$ is suspended from the ceiling, horizontally by means of two vertical wires of equal length tied to its ends. One of the wires is made of steel and its cross-section is $0.05 \mathrm{sq}. \mathrm{~cm}$ and the other is of brass of cross-section $0 \cdot 1 \mathrm{sq}. \mathrm{cm}$. Find the position along the rod at which a weight may be hung to produce AN 15 2021

(i) Equal stresses in both the wires,

(ii) Equal strain in both the wires.

Young's modulus of elasticity of brass and steel are $1.0 \times 10^{11} \mathrm{~N} / \mathrm{m}^2$ and $2.0 \times 10^{11} \mathrm{~N} / \mathrm{m}^2$ respectively.

75. Consider the diagram below with a water flow rate Q. Derive the expression for Q in terms of the difference in the manometer heights h and the cross-section areas A_1 and A_2 : TC 15 2022



76. Define moment of inertia and radius of gyration of a body of mass M rotating about an axis. State and prove Parallel Axis theorem on moment of inertia. TC 15 2022

Special Theory of Relativity

77. What is the significance of the null result of Michelson-Morley experiment? Does it disprove the existence of ether? Justify. TC 10 2010

78. A spaceship measures 50 m in length on ground and it measured a length of 49.7 m in space as observed from the ground. Find out the speed of the spaceship. AN 10 2011

79. Two identical relativistic particles of rest mass m and kinetic energy T collide head-on. What is the relative kinetic energy, i.e., the kinetic energy T' or one in the rest frame of the other? TC 12 2012

80. A projectile of mass M explodes, while in flight, into three fragments. One fragment of mass $m_1 = M/2$ travels in the original direction of the projectile. Another fragment of mass $m_2 = M/6$ travels in the opposite direction and the third fragment of mass $m_3 = M/3$ comes to rest. The energy E, released in the explosion, is 5 times the kinetic energy of the projectile at explosion. What are the velocities of the fragments? AN 15 2012

81. A particle of rest mass $\mathrm{M} = 4 \times 10^{-27}$ kg, disintegrates into two particles of rest masses $\mathrm{M}_1 = 3 \times 10^{-2}$ kg and $\mathrm{M}_2 = 1 \times 10^{-27}$ kg. Show that the energies E_1 and E_2 of these two parts after disintegration satisfy the condition $\mathrm{E}_1 = 3\mathrm{E}_2$ while moving in opposite directions with equal linear momenta. Give necessary mathematical derivation. AN 15 2013

82. Show that the operator $\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)$ is invariant under Lorentz transformations. TC 20 2013

83. Show that a particle of rest mass m_0, total energy E and linear momentum $\vec{p}$ satisfies the relation TC 10 2013

$$\mathrm{E}^2 = \mathrm{c}^2\mathrm{p}^2 + \mathrm{m}_0^2\mathrm{c}^4$$

where c is the velocity of light in free space.

84. Derive the relativistic length contraction using Lorentz transformation. TC 10 2013

85. A mirror is moving through vacuum with a relativistic speed v in the x-direction. A beam of light with frequency ω_i is normally incident (from $\mathrm{x} = \infty$) on the mirror. TC 25 2014

(i) What is the frequency of the reflected light expressed in terms of ω_{i}, c and v ?
(ii) What is the energy of each reflected photon?

86. Prove mathematically that the addition of any velocity of a particle to the velocity of light in free space merely reproduces the velocity of light in free space only. TC 10 2015

87. Show that the rest mass energy of an electron is 0.51 MeV. (use the standard values of the physical parameters). TC 10 2015

88. Calculate the percentage contraction in the length of a rod in a frame of reference, moving with velocity 0.8c in a direction AN 20 2016
   (i) parallel to its length and
   (ii) at an angle of 30° with its length.

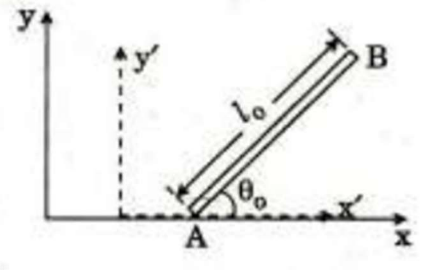
   What is the orientation of the rod in the moving frame of reference in case (ii)?

89. Given a proton for which $\beta = 0.995$ measured in the laboratory. What are the corresponding relativistic energy and momentum? Take, $m_p = 1 \cdot 67 \times 10^{-24}$ g. AN 10 2016

90. Describe Michelson-Morley experiment and show how the negative results obtained from this experiment were interpreted. TC 10 2017

91. Prove that $x^2 + y^2 + z^2 = c^2t^2$ is invariant under Lorentz transformation. TC 10 2017

92. A rod of length l_0 is kept at rest in $x'y'$ plane of its rest frame making an angle θ_0 with x' axis. What is the length and orientation of the rod in a laboratory frame (x, y) in which the rod moves to the right with velocity v ? TC 15 2018



93. i. A reference frame S' moves with respect to rest frame S with a uniform velocity ' v ' parallel to x-direction. Show from Lorentz transformation that two events simultaneous $(t_1 = t_2)$ at different. positions $(\mathrm{x}_1 \neq \mathrm{x}_2)$ in S frame are not in general simultaneous in S′ frame. AN 15 2019
   ii. The mean life of π meson is 2×10^{-8} s. Calculate the mean life of a meson moving with a velocity of 0.8c, where c is the velocity of light.

94.	Two β-particles A and B emitted by a radioactive source R travel in opposite directions, each with a velocity of $0.9\mathrm{c}$ with respect to the source. Find the velocity of B with respect to A (Here c is the velocity of light).	AN	15	2019
95.	What do you understand by length contraction? Calculate the percentage length contraction of a rod moving with a velocity 0.8 c in a direction at 60° with respect to its own length.	AN	15	2020
96.	Derive the relativistic expression for kinetic energy by considering mass variation with velocity. Hence, establish the relation between momentum (p) and energy (E) for a relativistic particle; $\frac{dE}{dp} = v$.	TC	20	2020
97.	An observer detects two explosions, one that occurs near him at a certain time and another that occurs 2 ms later 100 km away. Another observer finds that the two explosions occur at the same place. What time interval separates the explosions to the second observer?	AN	10	2021
98.	A body of mass m at rest splits into two masses m_1 and m_2 by an explosion. After the split the bodies move with a total kinetic energy T in opposite direction. Show that their relative speed is $\sqrt{2\mathrm{Tm}/\mathrm{m}_1\ \mathrm{m}_2}$.	TC	15	2021
99.	Two spaceships approach each other, both moving with same speed as measured by a stationary observer on the Earth. Their relative speed is 0.7 c. Determine the velocity of each spaceship as measured by the stationary observer on the Earth.	AN	15	2022

WAVES & OPTICS

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Syllabus

- ✓ Interference: Interference of light-Young’s experiment, Newton’s rings, interference by thin films, Michelson interferometer.
- ✓ Multiple beam interference and Fabry-Perot interferometer. Holography and simple applications
- ✓ Diffraction: Fraunhofer diffraction-single slit, double slit, diffraction grating, resolving power. Fresnel diffraction: – half-period zones and zones plates
- ✓ Fresnel integrals. Application of Cornu’s spiral to the analysis of diffraction at a straight edge and by a long narrow slit. Diffraction by a circular aperture and the Airy pattern
- ✓ Polarisation and Modern Optics: Production and detection of linearly and circularly polarised light. Double refraction, a quarter-wave plate
- ✓ Optical activity. Principles of fibre optics attenuation; pulse dispersion in step-index and parabolic index fibres; material dispersion, single-mode fibres. Lasers
- ✓ Einstein A and B coefficients. Ruby and He-Ne lasers. Characteristics of laser light-spatial and temporal coherence. Focussing on laser beams. Three-level scheme for laser operation.

Waves

1. In the propagation of longitudinal waves in a fluid contained in an infinitely long tube of cross-section A, show that TC 15 2010

$$\rho = \rho_o \left(1 - \frac{\partial \xi}{\partial x}\right)$$

where, $\rho_0 =$ equilibrium density
$\rho =$ density of the fluid in the disturbed state

$$\frac{\partial \xi}{\partial x} = \text{ volume strain } \left(\left|\frac{\partial \xi}{\partial x}\right| \ll 1\right)$$

2. Write down the one-dimensional harmonic oscillator differential equation under damping and its solution for the lightly damped condition, with the meanings of symbols. TC 10 2011
Determine the dependent energy in the lightly damped condition.

3. Explain the physical significance of group velocity from the concept of phase velocity with relevant expressions. TC 15 2011

4. Prove that the group velocity V_g of electromagnetic waves in a dispersive medium with refractive index $n(\lambda_0)$ at wavelength λ_0 is given by TC 20 2011

$$V_g = \frac{c}{n(\lambda_0) - \lambda_0 \dfrac{dn(\lambda_0)}{d\lambda_0}}$$

where c is the free space velocity of light. Find the time taken for the electromagnetic pulse to travel a distance D.

5. The motion of a damped mechanical oscillator is represented by TC 12 2012

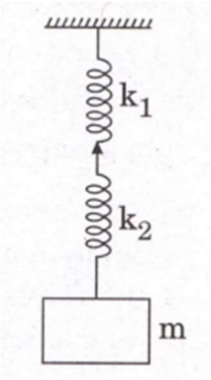
$$m\ddot{x} + \alpha\dot{x} + \beta x = 0$$

where m, α and β are constants. The oscillator is critically damped. The system is given an impulse at $x = 0$ and $t = 0$, resulting in an initial velocity v. After how much time the system experiences maximum displacement?

6. Show that a travelling wave on the string, clamped on both the ends, undergoes a phase change of π. Hence obtain the time-independent form of the wave equation representing a standing wave on the string. TC 12 2012

7. During an earthquake, a horizontal shelf moves vertically. If its motion can be regarded simple harmonic, calculate the maximum value of amplitude of oscillation so that the books resting on it stay in contact with it always. Take $\mathrm{g} = 9.8 \text{ ms}^{-2}$ and $\mathrm{T} = 0.5$ s. AN 10 2013

8.	The dispersion relation for deep water waves is given by $\omega^2 = \mathrm{gk} + \mathrm{ak}^3$ where g and a are constants. Obtain expressions for phase velocity and group velocity in terms of the wavelength λ. ω and k represent the angular frequency and wave number, respectively.	TC	5 10	2013
9.	The displacement associated with a three-dimensional plane wave is given by $\Psi(\mathrm{x},\mathrm{y},\mathrm{z},\mathrm{t}) = \mathrm{a}\cos\left[\frac{\sqrt{3}}{2}\mathrm{kx} + \frac{1}{2}\mathrm{ky} - \omega\mathrm{t}\right].$ Calculate the angles made by the propagating wave with the x, y and z-axes.	TC	10	2013
10.	In a certain engine, a piston undergoes vertical SHM with an amplitude of $10\ \mathrm{cm}$. A washer rests on the top of the piston. As the motor is slowly speeded up, at what frequency will the washer no longer stay in contact with the piston?	AN	10	2014
11.	Show that the group velocity is equal to particle velocity. Also prove that the group velocity of the photons is equal to c, the velocity of light.	TC	15	2014
12.	Find out the phase and group velocities of a radio wave of frequency $\omega = \sqrt{2}\omega_p$ in the ionosphere (as a dielectric medium) of refractive index $\mathrm{n} = \sqrt{1 - \frac{\omega_\mathrm{p}^2}{\omega^2}}$. Here, ω_p is the ionospheric plasma frequency.	AN	15	2015
13.	The equation of a progressive wave moving on a string is $y = 5\sin\ \pi(0 \cdot 01x - 2t)$. In this equation, y and x are in centimetres and t is in seconds. Calculate amplitude, frequency and velocity of the wave. If two particles at any instant are situated $200\ \mathrm{cm}$ apart, what will be the phase difference between these particles?	AN	10	2016
14.	Find the velocity of sound in a gas in which two waves of wavelengths $1.00\ \mathrm{m}$ and $1.01\ \mathrm{m}$ produce 10 beats in 3 seconds.	AN	10	2017
15.	When the two waves of nearly equal frequencies interfere, then show that the number of beats produced per second is equal to the difference of their frequencies.	TC	10	2018
16.	The equation for displacement (X) of a point on a damped oscillator is given by $x = 5e^{-0 \cdot 25t}\sin\left(\frac{\pi}{2}\right)t$ metres. Find the velocity of oscillating point at $t = \frac{T}{4}$ and T, where T is the time period of the oscillator. What is the direction of velocity in each case?	AN	10	2020

17.	A mass m is suspended by two springs having force constants k_1 and k_2 as shown in the figure. The mass m is displaced vertically downward and then released. If at any instant t, the displacement of the mass m is x, then show that the motion of the mass is simple harmonic motion having frequency $\mathrm{f} = \frac{1}{2\pi}\sqrt{\frac{1}{\mathrm{m}}\left(\frac{\mathrm{k}_1\mathrm{k}_2}{\mathrm{k}_1 + \mathrm{k}_2}\right)}$ 	TC	10	2021

Geometrical Optics

18.	What is damped harmonic oscillation? Write the equation of motion and obtain the general solution for this oscillation. Discuss the cases of dead beat, critical damping and oscillatory motion based on the general solution. What would be the logarithmic decrement of the damped vibrating system, if it has an initial amplitude $30\ \mathrm{cm}$, which reduces to $3\ \mathrm{cm}$ after 20 complete oscillations?	TC	20	2021
19.	Two thin symmetrical lenses of two different natures (convex and concave) and of different materials have equal radii of curvature $R = 15\ \mathrm{cm}$. The lenses are put close together and immersed in water $\mu_w = 4/3)$. The focal length of the system in water is $30\ \mathrm{cm}$. Show that the difference between the refractive indices of two lenses is $1/3$.	TC	10	2010
20.	Show that two convex lenses of the same material kept separated by a distance a, which is equal to the average of two focal lengths, may be used as an achromat, that is, $a = \frac{1}{2}(f_1 + f_2)$.	TC	10	2010
21.	Use matrix method to obtain an expression for the focal length of a coaxial combination of two thin lenses having focal lengths f_1 and f_2 separated by distance d.	TC	20	2012
22.	A convex lens of focal length $20\ \mathrm{cm}$ is placed after a slit of width $0.5\ \mathrm{mm}$. If a plane wave of wavelength 5000Å falls normally on the slit, calculate the	AN	10	2015

	separation between the second minima on either side of the central maximum.			
23.	Using matrix method, find out the equivalent focal length for a combination of two thin lenses of focal lengths f_1 and f_2 separated by a distance a.	TC	10	2015
24.	Obtain the system matrix for a thin lens placed in air and made of material of refractive index 1.5 having radius of curvature $50\ \mathrm{cm}$ each. Also find its focal length.	AN	10	2017
25.	What do you mean by spherical aberration of a lens? Show that if two plano-convex lenses are kept at a distance equal to the difference of their focal lengths, the spherical aberration would be minimum.	TC	15	2018
26.	What is axial chromatic aberration? A convex lens has a focal length of $15.5 \times 10^{-2}\ \mathrm{m}$ for red colour and $14.45 \times 10^{-2}\ \mathrm{m}$ for violet colour. If an object is kept at a distance of $40\ \mathrm{cm}$ from the lens, calculate the longitudinal chromatic aberration of the lens.	AN	10	2019
27.	Prove that when light goes from one point to another via a plane mirror, the path followed by light is the one for which the time of flight is the least.	TC	20	2019
28.	State and explain Fermat's principle of extremum path. Discuss the cases of rectilinear propagation of light and reversibility of light rays in context of Fermat's principle. Using Fermat's principle, deduce the thin lens formula.	TC	15	2020
29.	A thin film of petrol of thickness $9 \times 10^{-6}\ \mathrm{cm}$ is viewed at an angle 30° to the normal. Find the wavelength(s) of light in visible spectrum which can be viewed in the reflected light. The refractive index of the film $\mu = 1 \cdot 35$.	AN	10	2021
30.	What is chromatic aberration? Obtain the condition for achromatism using combination of two thin lenses placed in contact to each other. Can this system work as achromatic doublet if both are of same material? Justify your answer.	TC	15	2021
31.	Obtain the system matrix for a thick lens and derive the thin lens formula.	TC	20	2022

Interference

32.	An optical beam of spectral width $7.5\ \mathrm{GHz}$ at wavelength $\lambda = 600\ \mathrm{nm}$ is incident normally on Fabry-Perot etalon of thickness $100\ \mathrm{mm}$. Taking refractive index unity, find the number of axial modes which can be supported by the etalon.	AN	10	2010
33.	Describe Michelson interferometer for evaluation of coherence length of an optical beam. Calculate coherence length of a light beam of wavelength 600 nm with spectral width of 0.01 nm	AN	20 10	2010

34.	Show that two light beams polarized in perpendicular directions will not interfere.	TC	15	2010
35.	When a thin film of a transparent material is put behind one of the slits in Young's double-slit interference experiment, the zero-order fringe moves to the position previously occupied by the fourth-order bright fringe. The index of refraction of the film is $n = 1.2$ and the wavelength of light, $\lambda = 5000\text{Å}$. Determine the thickness of the film.	AN	10	2011
36.	The separation between the slits is 0.5 mm in Young's double-slit experiment. The interference pattern observed on a screen placed 5 m away reveals the location of the first maximum which is 6 mm from the centre of the pattern. Calculate the wavelength of light and separation between second and third bright fringes.	AN	12	2012
37.	In a Young double slit experiment, the first bright maximum is displaced by $\mathrm{y} = 2\ \mathrm{cm}$ from the central maximum. If the spacing between slits and distance from the screen are $0.1\ \mathrm{mm}$ and $1\ \mathrm{m}$ respectively, find the wavelength of light.	AN	10	2014
38.	In Michelson interferometer, 100 fringes cross the field of view when the movable mirror is displaced through $0.029\ \mathrm{mm}$. Calculate the wavelength of the light source used.	AN	5	2016
39.	Obtain the conditions for constructive interference and destructive interference in a thin film due to reflected light.	TC	15	2016
40.	Explain with proper example the interferences due to 'division of wavefront' and 'division of amplitude'.	TC	10	2017
41.	What is multiple-beam interference? Discuss the advantages of multiple-beam interferometry over two-beam interferometry. Explain the fringes formed by Fabry-Perot interferometer.	TC	15	2017
42.	What are the fringes of equal thickness and fringes of equal inclination? In a Newton's ring arrangement with a source emitting two wavelengths $\lambda_1 = 6 \times 10^{-7}\ \mathrm{m}$ and $\lambda_2 = 5.9 \times 10^{-7}\ \mathrm{m}$, it is found that the m^{th} dark ring due to one wavelength coincides with the $(m+1)^{th}$ dark ring due to the other. Find the diameter of the m^{th} dark ring, if the radius of curvature of the lens is $90\ \mathrm{cm}$.	AN	10	2019
43.	What are Newton's rings? How are they formed by two curved surfaces?	TC	10	2020
44.	Discuss the conditions for interference. Describe Young's double-slit experiment and derive an expression for the estimation of fringe width. Discuss its dependency on various parameters. Green light of wavelength 5100Å from a narrow slit is incident on a double-slit. If the overall separation of 10 fringes on a screen $200\ \mathrm{cm}$ away is $2\ \mathrm{cm}$, find the slit separation.	AN	20	2020

45. Newton's rings are observed between a spherical surface of radius of curvature 100 cm and a plane glass plate. The diameters of 4th and 15 th bright rings are 0.314 cm and 0.574 cm, respectively. Calculate the diameters of 24 th and 36th bright rings and also the wavelength of light used. TC 2022

Diffraction

46. Obtain the expression for the primary focal length of Fresnel zone plate. TC 20 2010

47. The Fraunhofer single-slit diffraction intensity is given by TC 10 2011

$$I = I_0 \frac{\sin^2 x}{x^2}$$

where $x = \frac{\pi d y}{\lambda l}$, with l as the distance from slit to source, d the slit width, y the detector distance and λ the wavelength. What is the value of cumulative intensity $\int_{-\infty}^{\infty} I(y) dy$?

48. In relation to a plane diffraction grating having 5000 lines per cm and irradiated by light of wavelength 6000Å, answer the following: AN 15 2011
(i) What is the highest order spectrum which may be observed?
(ii) If the width of opaque space is exactly twice that of the transparent space, which order of spectra will be absent?

49. Distinguish between Fresnel and Fraunhofer classes of diffraction. Show that the area of each Fresnel half-period zone is same. TC 20 2012

50. A diffraction grating of width 5 cm with slits of width 10^{-4} cm separated by a distance of $2 \times 10^{-4} \text{ cm}$ is illuminated by light of wavelength 550 nm. What will be the width of the principal maximum in the diffraction pattern? Would there be any missing orders? AN 20 2012

51. A parallel beam of light from a He - Ne laser $(\lambda = 630 \text{ nm})$ is made to fall on a narrow slit of width $0.2 \times 10^{-3} \text{ m}$. The Fraunhofer diffraction pattern is observed on a screen placed in the focal plane of a convex lens of focal length 0.3 m. Calculate the distance between the (i) first two minima and (ii) first two maxima on the screen. AN 15 2013

52. Explain the physical significance of resolving power of a grating with relevant mathematical expression. TC 10 2013

53. Considering a plane transmission diffraction grating, where d is the distance between two consecutive ruled lines, m as the order number and 0 as the TC 10 2014

	angle of diffraction for normal incidence, calculate the angular dispersion $\frac{d\theta}{d\lambda}$ for an incident light of wavelength λ.			
54.	Can D_1 and D_2 lines of sodium light $\left(\lambda_{D_1} = 5890\text{Å}\right.$ and $\left.\lambda_{D_2} = 5896\text{Å}\right)$ be resolved in second-order spectrum if the number of lines in the given grating is 450? Explain.	AN	10	2016
55.	Obtain an expression for the resolving power of a grating explaining the Rayleigh's criterion of resolution.	TC	15	2016
56.	Show that the areas of all the half-period zones are nearly the same. Find the radius of 1 st half-period zone in a zone plate whose focal length is $50\ \mathrm{cm}$ and the wavelength of the incident light is $500\ \mathrm{nm}$.	AN	15	2017
57.	A plane transmission grating has 3000 lines in all, having width of $3\ \mathrm{mm}$. What would be the angular separation in the first order spectrum of the two sodium lines of wavelengths 5890Å and 5896Å? Can they be seen distinctly?	AN	10	2018
58.	Discuss the intensity distribution in Fraunhofer diffraction pattern due to a single slit. Obtain conditions for maxima and minima of the intensity distribution. Show that the intensity of the first maxima is about 4.95% of that of the principal maxima.	TC	20	2018
59.	Show that the phenomenon of Fraunhofer diffraction at two vertical slits is modulation of two terms viz. double slit interference and single slit diffraction. Obtain the condition for positions of maxima and minima.	TC	20	2021
60.	Discuss the phenomenon of Fraunhofer diffraction at a single slit and show that the intensities of successive maxima are nearly in the ratio $1:\frac{4}{9\pi^2}:\frac{4}{25\pi^2}:\frac{4}{49\pi^2}$	TC	20	2022

Polarisation and Modern Optics

61.	An unpolarized light beam of intensity $1000\ \mathrm{W/m^2}$ is incident on an ideal linear polarizer with its transmission axis parallel to vertical direction. Describe an experiment to reduce the intensity of light beam to $500\ \mathrm{W/m^2}$.	AN	10	2010
62.	What should be the refractive index of cladding of an optical fibre with numerical aperture 0.5 with refractive index of core as 1.5?	AN	10	2010
63.	A laser beam of 1 micrometer wavelength with 3 megawatts power of beam diameter $10\ \mathrm{mm}$ is focussed by a lens of focal length $50\ \mathrm{mm}$. Evaluate the electric field associated with the light beam at the focal point. (Dielectric permittivity of free space, $\varepsilon_0 = 8 \cdot 8542 \times 10^{-12}\ C^2/N - \mathrm{m}^2$)	AN	25	2010

64.	A plane wave has the following expression for its electric field: $$\vec{E} = \hat{x}E_{0x}\cos(\omega t - kz + \alpha) + \hat{y}E_{0y}\cos(\omega t - kz + \beta)$$ If the phase difference is defined as $\delta = \beta - \alpha$, under what conditions do we achieve elliptic polarization? What are the conditions for circular polarization?	TC	10	2011
65.	For calcite, the refractive indices of ordinary and extraordinary rays are 1.65836 and 1.48641 at $\lambda_0 = 5893\text{Å}$ respectively. A left circularly polarized beam of this wavelength is incident normally on such crystal of thickness $0.005141\ \text{mm}$ having its optic axis cut parallel to the surface. What will be the state of polarization of the emergent beam?	AN	15	2011
66.	Bring out the essential differences between the physical principles of spontaneous and stimulated emission of radiation. Why is it difficult to get efficient lasing action in case of an ideal two-level material system? Can you propose a scheme to enhance efficiency? Discuss.	TC	15	2011
67.	Show with proper mathematical analysis that the ratio of Einstein's A and B coefficients depends upon the energy separation between the two energy levels participating in the optical transitions. What is the physical significance of A coefficient? Justify the statement; "It is very difficult to develop an X-ray laser".	TC	20 5 5	2011
68.	Derive an expression for intermodal dispersion for a multimodal step-index fibre.	TC	20	2012
69.	A pulse of $\lambda_0 = 600\ \text{nm}$ and $\Delta\lambda = 10\ \text{nm}$ propagates through a fibre which has a material dispersion coefficient of 50ps per km per nm at $600\ \text{nm}$. Calculate the pulse broadening in traversing a $10\ \text{km}$ length of the fibre. If the pulse width at the input of the fibre is $12\ \text{ns}$, what will be the pulse width at the output of the fibre?	AN	10	2012
70.	Calculate the minimum thickness of a quartz plate which would behave as a quarter-wave plate for wavelength of light, $\lambda = 6000\text{Å}$. The refractive indices for ordinary and extraordinary rays are $\mu_o = 1.544$ and $\mu_e = 1.553$.	AN	15	2012
71.	Explain why information carrying capacity of an optical fibre can be enhanced by reducing the pulse dispersion. How does one minimize pulse dispersion using a graded index optical fibre?	TC	10	2013
72.	What is the physical significance of Einstein's A-coefficient? Explain why it is more difficult to achieve Lasing action at X-ray wavelength than at infra-red wavelength.	TC	10	2014
73.	For a multimode step index optical fibre, the core refractive index is 1.5 and fractional index difference is 0.001. Calculate the pulse broadening for $1\ \text{km}$ length of the fibre. Over a length of $2\ \text{km}$ of the fibre, calculate the minimum pulse scparation that can be transmitted without overlap.	AN	10	2014

74.	Explain the working principle of a 3-level laser with a specific example. Comment on why the third level is needed.	TC	10	2014
75.	How does holography differ from conventional photography? What are the requirements for the formation and reading of a hologram?	TC	10	2014
76.	What is the role of an optical resonator in a laser? Why does one prefer curved mirrors instead of plane mirrors in designing an optical resonator?	TC	10	2015
77.	Find out the angle between the reflected and refracted rays when a parallel beam of light is incident on a dielectric surface at an angle equal to the Brewster's angle. Explain how do you use this concept to produce linearly polarized light.	TC	10 10	2015
78.	Using the concept of Einstein's A and B coefficients for a two-level atomic system under thermal equilibrium, determine the ratio of the number of atoms per unit volume in the two levels experiencing spontaneous and stimulated emission. How does the principle of population inversion lead to the gain mechanism in the active medium of the laser?	TC	10 10	2015
79.	The refractive indices of core and cladding in a step index optical fiber are 1.52 and 1.48 respectively. The diameter of the core is $30\mu\text{m}$. If the operating wavelength is $1.3\mu\text{m}$, calculate the V parameter and the maximum number of modes supported by the fiber.	AN	10	2016
80.	Explain the principle of (i) induced absorption, (ii) spontaneous emission and (iii) stimulated emission. Show that the ratio of Einstein's coefficients is given by $\frac{A}{B} = \frac{8\pi h v^3}{c^3}$	TC	20	2016
81.	Explain the principle of producing polarized light by the method of reflection, refraction and double refraction with the help of neat diagrams.	TC	15	2016
82.	Sunlight is reflected from a calm lake. The reflected light is 100% polarized at a certain instant. What is the angle between the sun and horizon?	AN	10	2017
83.	A plane-polarized light passes through a double-refracting crystal of thickness $40\mu\text{m}$ and emerges out as circularly polarized. If the birefringence of the crystal is 0.00004, then find the wavelength of the incident light.	AN	10	2017
84.	How is laser light different from ordinary light? Discuss the working principle of ruby laser. What role do chromium ions play in this process?	TC	15	2017
85.	Explain the principle of operation of optical fibre. What are the different losses that take place in optical fibre?	TC	10	2017
86.	Explain the principle and working of He-Ne laser. What is the role of He gas? Why is it necessary to use narrow tube? How many longitudinal modes can be excited for an He-Ne laser in a cavity of length 30 cm and having half	AN	15	2018

width of gain profile of laser material 2×10^{-3} nm ? The emission wavelength is 6328Å.

87. Distinguish between positive and negative crystals in terms of double refraction. How are these crystals used to make quarter wave plates? Explain how the quarter wave plate is used in producing elliptically and circularly polarized light. TC 15 2018

88. How can one convert a left-handed circularly polarised light into a right-handed one (and vice versa)? AN 10 2019

Calculate the thickness of a quarter-wave plate when the wavelength of light is $589\ \mathrm{nm}$.
Given: $\mu_0 = 1.544$ and $\mu_E = 1.553$.

89. Discuss how population inversion is achieved in Ruby laser. TC 20 2019
What is 'laser spiking'? Why does it occur?

90. In what way is holography different from conventional photography? Discuss the salient features of a hologram. What are the requirements for the formation and reading of a hologram? TC 20 2019

91. What is a zone plate? Give its theoretical description. Show that a zone plate has multiple foci. Differentiate a zone plate from a convex lens. Calculate the radius of the first half period zone in a zone plate behaving like a convex lens of focal length $60\ \mathrm{cm}$ for light of wavelength 6000Å. AN 15 2020

92. Briefly discuss the postulates of Einstein to explain stimulated emission. Derive an expression for Einstein's A and B coefficients and show that the ratio of coefficients of spontaneous versus stimulated emission is proportional to the third power of frequency of radiation. Why is it difficult to achieve laser action in higher frequency ranges such as X-rays? TC 15 2020

Can there be a temperature at which the rates of spontaneous and stimulated emission are equal? Illustrate with wavelength $\lambda = 5000\text{Å}$.

93. Explain the phenomenon of double refraction in calcite crystal. Considering birefringent crystal as non-conducting material, explain double refraction using electromagnetic theory. AN 15 2020

Calculate the thickness of a double refracting plate which produces a path difference of $\frac{\lambda}{4}$ between extraordinary and ordinary waves.
Given:

$$\lambda = 5890\text{Å}, \mu_0 = 1.53, \mu_\mathrm{e} = 1.54$$

94. In a step-index optical fiber system, explain the terms pulse dispersion and material dispersion. AN 20 2021

An optical fiber having refractive indices of core and cladding $n_1 = 1.463$ and $n_2 = 1.444$ respectively, uses a Laser diode with $\lambda_0 = 1.50\mu\mathrm{m}$ with a spectral width of $2\ \mathrm{nm}$. At this wavelength if the material dispersion

	coefficient, $\mathrm{D_m}$ is $18.23\mathrm{ps/km.nm}$, then calculate the pulse dispersion and material dispersion for $1\ \mathrm{km}$ length of the fiber.			
95.	A phase retardation plate of quartz has thickness $0.1436\ \mathrm{mm}$. For what wavelength in the visible region will it act as quarter-wave plate? Given that $\mu_{\mathrm{O}} = 1.5443$ and $\mu_{\mathrm{E}} = 1.5533$.	AN	10	2022
96.	In He-Ne laser, what is the function of He gas? Explain the answer with the help of energy level diagram for $\mathrm{He} - \mathrm{Ne}$ laser.	TC	15	2022

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Syllabus

(a) Electrostatics and Magnetostatics:

- ✓ Laplace and Poisson equations in electrostatics and their applications;
- ✓ Energy of a system of charges, multiple expansion of scalar potential;
- ✓ Method of images and its applications;
- ✓ Potential and field due to a dipole, force and torque on a dipole in an external field;
- ✓ Dielectrics, polarization; Solutions to boundary-value problems-conducting and dielectric spheres in a uniform electric field;
- ✓ Magnetic shell, uniformly magnetized sphere; Ferromagnetic materials, hysteresis, energy loss.

(b) Current Electricity:

- ✓ Kirchhoff's laws and their applications;
- ✓ Biot-Savart law, Ampere's law, Faraday's law, Lenz' law; Self-and mutual-inductances;
- ✓ Mean and r m s values in AC circuits; DC and AC circuits with R, L, and C components;
- ✓ Series and parallel resonances; Quality factor; Principle of transformer.

Electrostatics and Magnetostatics

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1. Obtain Poisson's equation in electrostatics from Gauss' law. What form does it take when the charge density is zero? TC 10 2010

2. What is meant by a dielectric? Define polarization vector P and relate it with the average molecular dipole moment. Obtain expression for the potential due to a polarized dielectric in terms of the polarization vector. TC 20 2010

3. Find out the total electric potential energy of a single spherical object of uniform charge density p, total charge Q and radius R. TC 10 2011

4. Determine the torque experienced by an electric dipole of moment $\vec{p}$ if placed in an electric field $\vec{E}$ in nonaligned state. Also show that the interaction energy of two dipoles of moments $\vec{p}_1$ and $\vec{p}_2$ separated by a displacement $\vec{r}$ is AN 20 2011

$$U = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^3}[\vec{p}_1 \cdot \vec{p}_2 - 3(\vec{p}_1 \cdot \hat{r})(\vec{p}_2 \cdot \hat{r})]$$

assuming the expression for field due to a dipole.

5. The electric potential of a grounded conducting sphere of radius a in a uniform electric field is given as AN 10 2011

$$\phi(r,\theta) = -E_0 r\left[1 - \left(\frac{a}{r}\right)^3 \cos\,\theta\right]$$

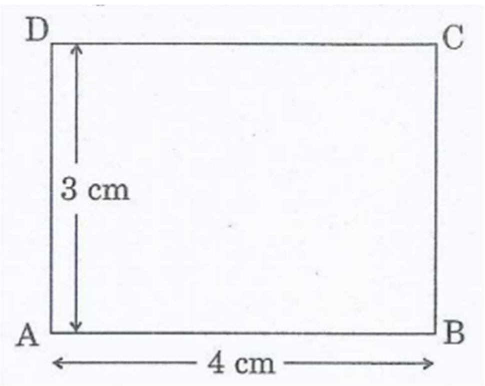
Find the surface charge distribution.

6. The volume between two concentric conducting spherical surfaces of radii a and $b(a < b)$ is filled with an inhomogeneous dielectric with $\varepsilon = \varepsilon_0/(1 + cr)$, where c is a constant and r is the radial coordinate. A charge $+Q$ is placed on the inner surface, while the outer surface is grounded. Determine- AN 15 2012
15
15
15

(a) $\vec{D}$ in the region $a < r < b$;
(b) capacitance of the device;
(c) polarization charge density in the region $a < r < b$;
(d) surface polarization charge densities at $r = a$ and $r = b$.

7. Assume $\vec{E} = 0$ inside a perfect conductor. Elaborate on any other four electrostatic properties that arise from this property. TC 15 2012

8. Using the fundamental concepts of electromagnetism, determine the electric field of an electric dipole $\vec{p}$ at a distance $\vec{r}$ and its energy in an electric field $\vec{\mathrm{E}}$. TC 15 2013

9.	$ABCD$ is a rectangle in which charges of $+10^{-11}\mathrm{C}, -2 \times 10^{-11}\mathrm{C}$ and $10^{-11}\mathrm{C}$ are placed at corners B, C and D, respectively.  Calculate the potential at the corner A and the work done in carrying a charge of 2 coulombs to A.	AN	10	2013
10.	Under one-dimensional configuration, the charge density is given by $\rho(x) = \frac{\rho_0 x}{5}$; where ρ_0 is a constant charge density. If the electric field $\lvert\vec{\mathrm{E}}\rvert = 0$ at $\mathrm{x} = 0$ and potential $\mathrm{V} = 0$ at $\mathrm{x} = 5$, determine V and $\lvert\vec{\mathrm{E}}\rvert$.	AN	10	2015
11.	A conducting sphere of radius $5\ \mathrm{cm}$ has a total charge of $12\mathrm{nC}$ uniformly distributed on its surface in free space. Determine the displacement vector $\vec{D}$ on its surface and outside at a distance r from the centre of the sphere.	AN	10	2015
12.	With the help of a neat diagram, show that the potential due to a dipole at a point is given by $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p\cos\,\theta}{r^2}$, where p is the dipole moment of the charge distribution, θ is the angle between the line joining the centre of the dipole to the point of interest and the axis of the dipole.	TC	10	2016
13.	Discuss the principle of 'artificial dielectric'. Where do you find its use?	TC	10	2017
14.	A charge $q = 2\mu\mathrm{C}$ is placed at $a = 10\ \mathrm{cm}$ from an infinite grounded conducting plane sheet. Find the (i) total charge induced on the sheet, (ii) force on the charge q and (iii) total work required to remove the charge slowly to an infinite distance from the plane.	AN	10	2017
15.	A current $i(t) = (2e^{-t} - e^{-2t})\mu\mathrm{A}$ charges up a $120\mathrm{nF}$ capacitor for a period of 2 seconds. If the final voltage across the capacitor is $15\ \mathrm{V}$, what was the initial voltage across it?	AN	10	2017
16.	Why does a soap bubble expand upon electrification? A sphere of radius R contains a charge $+Q$ and a charge $-Q$ distributed uniformly in the upper and lower hemispheres respectively. Show that the dipole moment of charge distribution is $\frac{3}{4}QR\hat{k}$, where $\hat{k}$ is directed along the polar axis of the spherical coordinate system.	TC	15	2017

17. Discuss briefly the features of 'guard rings'. TC 15 2017

The plates of a capacitor are square-shaped, each of side l. The plates are inclined at an angle α to each other. The smallest distance between the plates is a. Calculate the capacitance when α is small.

18. A $0.5\ \mathrm{m}$ long cylindrical medium between two conducting plates has uniform charge density of $100\mathrm{nC/m^3}$. The axis of the cylindrical medium is along z-axis. The left plate is at $\mathrm{z} = 0$ and has a potential of $10\mathrm{kV}$ and the right plate is grounded. Determine the electric field at axial distance $\mathrm{z} = 0.2\ \mathrm{m}$. AN 15 2018

19. A uniformly magnetized sphere of radius R has magnetization $\vec{M} = M_0\hat{Z}$. If the scalar magnetic potentials inside and outside the sphere are given as under AN 15 2018

$$\phi_m = \frac{M_0}{3}Z; r \leq R$$

$$\text{and } \phi_m = \frac{M_0}{3}\frac{R^3}{r^2}\cos\,\theta; r > R$$

where, r, θ are two spherical coordinates, find the magnetic field inside and outside the sphere.

20. Starting from the expression for the electrostatic potential TC 10 2019

$$\phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho(\vec{r}_0)}{|\vec{r} - \vec{r}_o|}dV_o$$

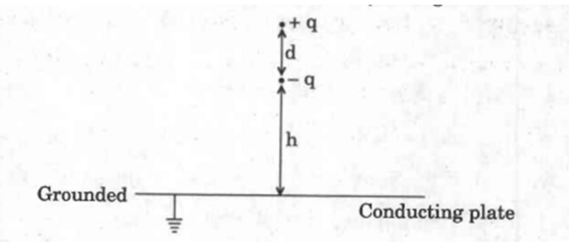
obtain Poisson's equation $\nabla^2\phi = -\frac{\rho}{\varepsilon_0}$.

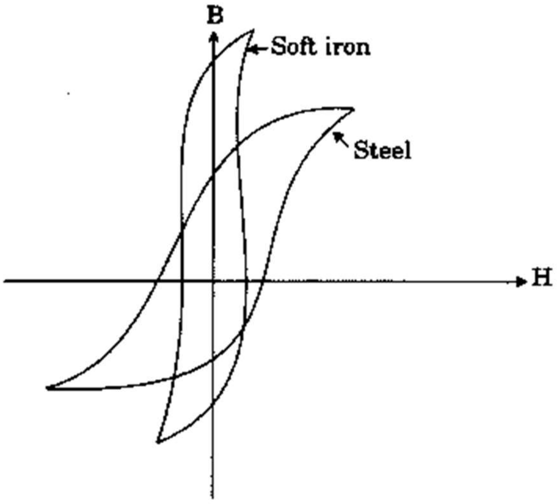
[Symbols have their usual meanings]

21. Find the capacitance of two concentric spherical metal shells having radii a and b. TC 10 2019

22. Two conducting planes, intersecting at right-angles to each other, are kept at a potential ϕ_0. Calculate the potential at a point in space if the total charge on a plane of area α be Q. TC 15 2019

23. A vertically oriented electric dipole having dipole moment $\vec{p}$ is kept at height h above an infinitely large horizontal conducting plate, which is grounded as shown in the diagram. Calculate the force between the electric dipole and the conducting plate by using method of images. TC 10 2020

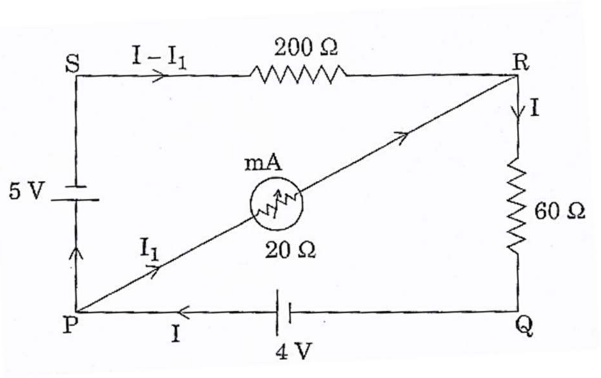


24.	Based on the hysteresis loops for soft iron and steel as shown in the diagram, which material would you prefer to utilise for making transformer cores and why?  	TC	10	2020
25.	Write expressions for divergence and curl of an electrostatic field. From these, obtain Poisson and Laplace equations. Two concentric conducting spherical shells having radii r_1 and $r_2 (r_1 < r_2)$ are charged to potentials V_1 and V_2, respectively. What are the electric potential and hence electric field in the space between the shells? Also find the charge on the inner shell.	TC	15	2020
26.	Given that the electric potential of a system of charges is $V = \frac{12}{r^2} + \frac{1}{r^3}$ volt. Calculate the electric field vector at the Cartesian point $(4,2,3)\mathrm{m}$.	AN	10	2021
27.	Given an infinite line charge of charge density $2\mathrm{nCm}^{-1}$ parallel to the y-axis and passing through the point $(3,0,4)\mathrm{m}$ and an infinite sheet of charge of charge density $4\mathrm{nCm}^{-2}$ parallel to the $\mathrm{x} - \mathrm{y}$ plane and passing through the point $(0,0,6)\mathrm{m}$. Calculate the electric field intensity at the point $(10,10,10)\mathrm{m}$. Assume free space.	AN	15	2021
28.	Consider two point particles of charge q each, separated by a distance d, and travelling at non-relativistic velocity $\vec{v}$. If the line joining the two charges is perpendicular to $\vec{v}$, then write an expression for the magnetic force between the two particles, and illustrate the direction of the force on each particle.	TC	10	2022
29.	Starting from the Laplace's equation in a cylindrical polar coordinate system and using the method of separation of variables, obtain the differential equations for the solutions of r, ϕ and z components of the potential.	TC	15	2022

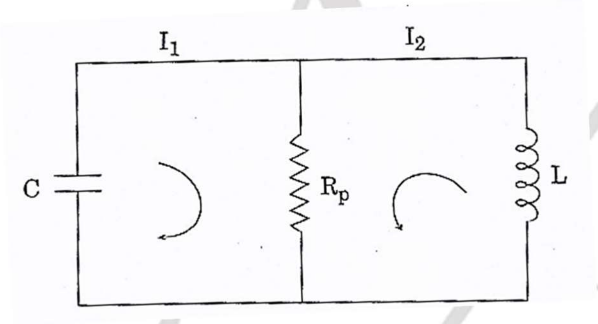
Current Electricity

30.	What happens if the primary winding of a transformer is connected to a battery?	TC	10	2010
31.	Discuss the growth of current when an e.m.f. is suddenly applied to a circuit containing resistance, inductance and capacitance in series. What is the time constant of the circuit?	TC	20	2010
32.	A series circuit has an inductance of 200 microhenries, a capacitance of 0.0005 microfarad and a resistance of 10 ohms. Find the resonant frequency and quality factor of the circuit.	AN	20	2010
33.	Find whether the discharge of a condenser through the inductive circuit is oscillatory when $C = 0.1\mu\text{F}$, $L: = 10\text{mH}$ and $R = 200\Omega$. If it is oscillatory, calculate its frequency.	AN	10	2011
34.	The four arms of a Wheatstone bridge have the following resistances: $AB = 100\Omega, BC = 10\Omega, CD = 5\Omega, DA = 60\Omega$ A galvanometer of 15Ω resistance is connected across BD. Calculate the current through the galvanometer when a potential difference of 10 volts is maintained across AC.	AN	10	2011
35.	A long solenoid has 220 turns $/\text{cm}$; its diameter is $3.2\ \text{cm}$. Inside the solenoid at its centre, we place a 130-turn closepacked coil of diameter $2.1\ \text{cm}$ along its axis. The current in the solenoid is increased from zero to 1.5 amperes at a steady rate over a period of 0.16 second. What is the magnitude of the induced e.m.f. that appears in the central coil when the current in the solenoid is being changed?	AN	10	2011
36.	An electrical circuit consists of a resistance R, inductance L and capacitance C in series. If a charge is put on the capacitor at some instant, determine the condition that V_C, the voltage across the capacitor, is subsequently oscillatory. Derive an expression for the quality factor Q of the circuit by considering the decay of the oscillation, using the result that the amplitude falls by a factor of e in $\left(\frac{Q}{\pi}\right)$ period.	TC	20	2011
37.	A resistor $R(= 6.2\text{M}\Omega)$ and a capacitor $C(= 2.4\mu\text{F})$ are connected in series and a $12\ \text{V}$ battery of negligible internal resistance is connected across their combination. (i) What is the capacitive time constant of this circuit? (ii) At what time, after the battery is connected, does the potential difference across the capacitor become $5.6\ \text{V}$?	AN	5+5	2011
38.	A resistance R and a lossless capacitor C are connected through a switch. The capacitor is charged to potential V_0, and the switch is closed at $t = 0$. Prove that the energy stored in the capacitor is equal to the energy dissipated in the resistor.	TC	12	2012

39. In the circuit diagram shown below, calculate the current passing through the milliammeter. AN 10 2013



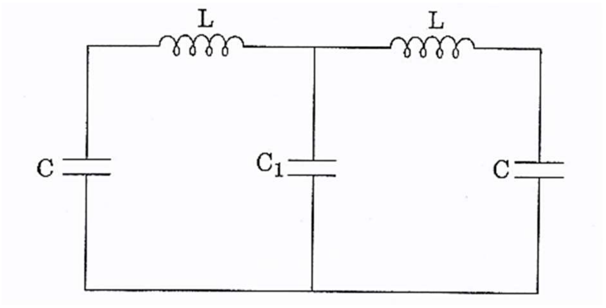
40. Consider the equation for a series RLC circuit and compare this to the parallel resonant circuit shown below: TC 10 2013



Calculate the value of R_p if a series RLC circuit and the parallel RLC circuit are to have same equations for the potential of capacitance while they both have the same L, C and Q with Q being the total charge.

41. A series LCR circuit has resonant frequency ω_0 and a large quality factor Q. Write down in terms of $\mathrm{R}, \omega, \omega_0$ and Q, its (i) impedance at resonance, (ii) impedance at half-power points and (iii) the approximate forms of its impedance at low and high frequencies. TC 15 2013

42. Consider the following coupled inductor - capacitor circuit: TC 10 2013



Calculate the ratio of the frequencies of the anti-symmetric and symmetric modes ω_a/ω_s.

$$\left(\text{Given } \mathrm{k}=\frac{1}{\mathrm{LC}}, \mathrm{k}'=\frac{1}{\mathrm{LC_1}}\right)$$

43. For initial current conditions $I = I_0$ and $\frac{dI}{dt} = 0$ at $t = 0$, show that the time dependent current in the critical damping case for an LCR circuit is given by AN 20 2014

$$I = I_0\left(1+\frac{\gamma t}{2}\right)e^{-\gamma t/2}$$

where $\gamma = \frac{R}{\mathrm{L}}$, $\omega_0^2 = \frac{1}{\mathrm{LC}}$, $\omega = \sqrt{\omega_0^2 - \frac{\gamma^2}{4}}$ and $\tan\ \delta = \frac{-\gamma}{2\omega}$.

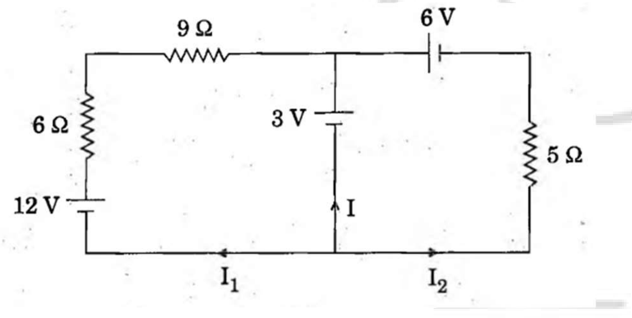
44. Using Ampere's Law and continuity equation, show that the divergence of the total current density is zero. TC 15 2014

45. When connected in series, L_1, C_1 have the same resonant frequency as $\mathrm{L_2}, \mathrm{C_2}$ also connected in series. Prove that if all these circuit elements are connected in series, the new circuit will have the same resonant frequency as either of the circuits first mentioned. TC 15 2014

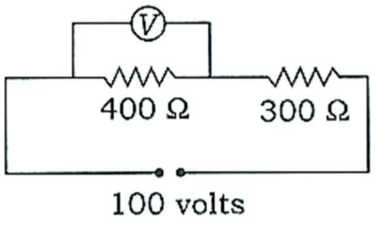
46. A series RLC circuit has a resistance of 100Ω and an impedance of 210Ω. If this circuit is connected to an a.c. source with an r.m.s. voltage of 220 V, how much is the average power dissipated in the circuit? AN 10 2015

47. A series RLC circuit has $R = 2\Omega$. The energy stored in the circuit decreases by 1% per period of oscillation. Its natural undamped frequency is 2kHz. Determine the values of inductor L and the quality factor. AN 15 2015

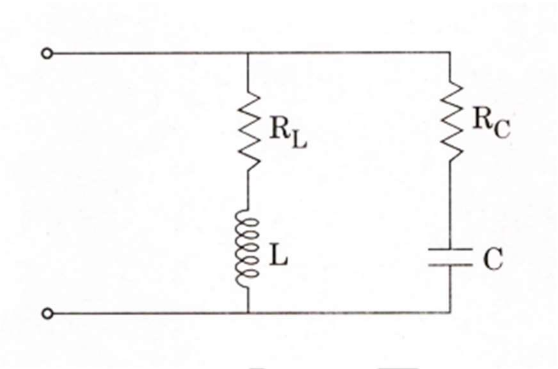
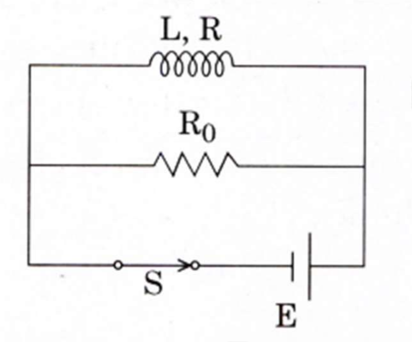
48. In the circuit given below, find the values of currents I_1, I_2 and I. AN 15 2015

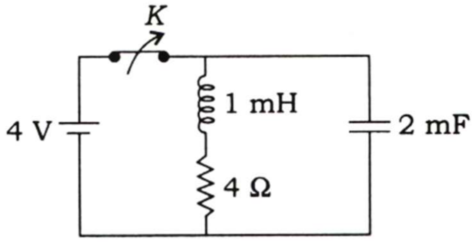


49. In the circuit diagram shown below, the voltmeter reads 50 volts when it is connected across the 400Ω resistance. Calculate what the same voltmeter will read when connected across the 300Ω resistance. AN 10 2016



50.	An alternating current varying sinusoidally with a frequency of $50\ \mathrm{Hz}$ has an r.m.s. value of 40 A. Find the instantaneous value of the current at 0.00125 second after passing through maximum positive value.	AN	10	2016
51.	How large an inductance needs to be connected in series with a $120\ \mathrm{V}, 60\ \mathrm{W}$ lightbulb if it is to operate normally when the combination is connected across a $240\ \mathrm{V}, 60\ \mathrm{Hz}$ supply?	AN	10	2017
52.	When a person carrying something metallic walks through the doorway of a metal detector, it emits a sound. Explain the reason behind it. A 200Ω resistor and a $15\mu\mathrm{F}$ capacitor are connected in series to $220\ \mathrm{V}, 50\ \mathrm{Hz}$ a.c. supply. Calculate the current in the circuit and the r.m.s. voltage across the resistor and the capacitor. Is the algebraic sum of these voltages more than the supply voltage? If yes, resolve the paradox.	AN	15	2017
53.	A current carrying circular wire loop of radius $1.0\ \mathrm{cm}$ has a magnetic moment $2.0\ \mathrm{mJ/T}$. Determine the magnetic field at an axial distance of $3.0\ \mathrm{cm}$ from the centre of the loop.	AN	10	2018
54.	A $12.0\ \mathrm{V}$ battery is connected at $\mathrm{t} = 0$ to a series combination of a resistor $R = 10.0\Omega$ and anductor $L = 5.0\mathrm{H}$. At what rate is energy being stored in the inductor when the current in the circuit is $0.4\ \mathrm{A}$?	AN	10	2018
55.	Two solenoids have 500 and 800 turns of wire and are placed co-axially close to each other. A current of 5.0 A in the first solenoid produces an average flux of $200\mu\mathrm{Wb}$ through its each turn and a flux of $100\mu\mathrm{Wb}$ through each turn of the second solenoid. Find the self-inductance of the first solenoid and the mutual inductance of the solenoids.	AN	10	2018
56.	Why do we prefer to work with a critically damped ballistic galvanometer in a laboratory? What is external critical damping resistance?	TC	10	2019
57.	Three cells are connected in parallel with similar poles connected together with wires having negligible resistance. The emfs of the cells are 2,1 and 4 volts respectively and the corresponding internal resistances are 4, 3 and $2\ \mathrm{ohms}$. Calculate the current flowing through the $4\ \mathrm{V}$ cell.	AN	15	2019
58.	Describe the oscillations of electric and magnetic fields in an ideal LC circuit. The applied voltage phasor in a circuit is $(4 + 3i)$ volt and resulting current phasor is $(3 + 4i)$ ampere. Draw the phasor diagram. Determine the impedance of the circuit and indicate whether it is inductive or capacitive in nature. Also find the power dissipation in the circuit.	TC	15	2020
59.	A 10Ω resistor is connected in series with a capacitor of $1.0\mu\mathrm{F}$ and a battery with emf $12.0\ \mathrm{V}$. Before the switch is closed at time $t = 0$, the capacitor is uncharged. Calculate the following: (i) The time constant. (ii) What fraction of the final charge is on the plates at the time $\mathrm{t} = 46$ seconds? (iii) What fraction of the initial current remains at the time $\mathrm{t} = 46$ seconds?	AN	15	2020

	Consider that the internal resistance of the battery is zero and neglect the resistance of all the connecting wires.			
60.	A rod of length l is perpendicular to a uniform magnetic field B. The rod revolves at an angular speed ω about an axis passing through one end of the rod and parallel to the magnetic field B. Find the voltage induced across the rod's ends.	TC	10	2021
61.	Consider the two branch parallel circuit shown in the diagram. Determine the resonant frequency of the circuit.  	TC	10	2021
62.	In the given circuit, $\mathrm{L} = 2.0\mu\mathrm{H}, \mathrm{R} = 1.0\Omega, \mathrm{R}_0 = 2.0\Omega$ and $\mathrm{E} = 3.0\ \mathrm{V}$. Find the amount of heat generated in the coil after the switch S is disconnected. The internal resistance of the source is negligible.  	AN	10	2021
63.	A cell of internal resistance $1\mathrm{ohm}, 1.5$ volt e.m.f. and another cell of internal resistance $2\mathrm{ohm}, 2$ volt e.m.f. are connected in parallel across the ends of an external resistance of $5\mathrm{ohm}$. Find the current in each branch of the circuit.	AN	10	2022
64.	Consider the R-L-C circuit shown here. Calculate the Q-factor of the circuit. Does the circuit have a resonant frequency? Justify your answer:	AN	10	2022


K
4 V
1 mH
4 Ω
2 mF

ELECTROMAGNETIC WAVES AND BLACKBODY RADIATION

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OCTOBER 3, 2023
ABHI PHYSICS
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Syllabus

- ✓ Displacement current and Maxwell's equations;

- ✓ Wave equations in vacuum, Pointing theorem; Vector and scalar potentials; Electromagnetic field tensor, covariance of Maxwell's equations;

- ✓ Wave equations in isotropic dielectrics, reflection and refraction at the boundary of two dielectrics; Fresnel's relations;

- ✓ Total internal reflection; Normal and anomalous dispersion; Rayleigh scattering;

- ✓ Black body radiation and Planck's radiation law, Stefan - Boltzmann law, Wien's displacement law and Rayleigh-Jeans' law.

Electromagnetic Theory

1.	A wire of length $2\ \mathrm{m}$ is perpendicular to $X-Y$ plane. It is moved with a velocity $\vec{V}=(2\vec{\imath}+3\vec{\jmath}+\vec{k})\mathrm{ms}^{-1}$ through a region of uniform induction $\vec{B}=(\vec{\imath}+2\vec{\jmath})\mathrm{Wm}^{-2}$. Compute the potential difference between the ends of the wire.	AN	10	2010
2.	Using Maxwell's field equations for a homogeneous non-conducting medium, derive the wave equation for the electric field. Calculate the velocity of EM wave in free space.	TC	20	2010
3.	Explain the term 'Poynting vector' and state the significance of Poynting theorem.	TC	20	2010
4.	Calculate the skin depth for radio waves in free space of wavelength $3\ \mathrm{m}$ in copper, given that electrical conductivity for copper is $6\times10^{7}\Omega^{-1}\ \mathrm{m}^{-1}$.	AN	20	2010
5.	Consider Maxwell's equation in differential form in media. For $j=\rho=0$, assume $\varepsilon=\varepsilon_0 e^{\alpha t}$ and $\mu=\mu_0 e^{\alpha t}$ and show that the relevant wave equation for a plane wave propagating along x-direction is $\frac{\partial^2 E}{\partial x^2}=\mu\frac{\partial^2 D}{\partial t^2}+\mu\alpha\frac{\partial D}{\partial t}$ where $\vec{E}=E\hat{y}$ and $\vec{H}=H\hat{z}$	TC	10	2011
6.	Consider a plane wave travelling along the positive y-direction incident upon a glass of refractive index $n=1.6$. Find the transmission coefficient.	AN	25	2011
7.	Justify which of the four Maxwell's equations imply that there are no magnetic monopoles. How these equations would have been written if they were?	TC	15	2011
8.	What is a displacement current? Prove that lines of conduction current plus displacement current are continuous.	TC	15	2012
9.	The electric field of a plane e.m. wave travelling along the z-axis is $\vec{\mathrm{E}}=\left(\mathrm{E}_{\mathrm{ox}}\hat{\mathrm{x}}+\mathrm{E}_{\mathrm{oy}}\hat{\mathrm{y}}\right)\sin\left(\omega t-\mathrm{kz}+\phi\right).$ Determine the magnetic field.	AN	10	2013
10.	(i) Considering an isotropic, linear, non-conducting, non-magnetic and inhomogeneous dielectric medium with $\vec{\mathrm{D}}=\varepsilon\vec{\mathrm{E}}=\varepsilon_0\mathrm{n}^2(\mathrm{x},\mathrm{y},\mathrm{z})\vec{\mathrm{E}},$	TC	15 5 5	2013

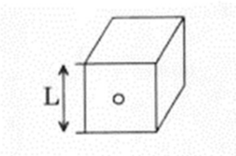
	show that the electromagnetic wave equation for the field $\vec{E}$ is given by $\nabla^2\vec{E} + \vec{\nabla}\left(\frac{1}{n^2}\vec{\nabla}n^2 \cdot \vec{E}\right) - \mu_0\varepsilon_o n^2 \frac{\partial^2\vec{E}}{\partial t^2} = 0.$ (ii) Write down the scalar equation for E_x from the above equation. (iii) Interpret physically the situation if we move from homogeneous to an inhomogeneous medium. (iv) Obtain the similar vector equation for the magnetic field $\vec{H}$ in inhomogeneous medium.		15	
11.	For a uniform wire of length L and radius a having a potential difference V between the ends and a current I along it, calculate the energy per unit time delivered to the wire by Poynting vector.	TC	10	2013
12.	Starting from Maxwell's equation, obtain the wave equation for the electric field $\vec{E}$ in free space and appropriate wave equation for the electric field $\vec{E} = E_z(x, y, z)\hat{z}$.	TC	10	2014
13.	Show that the energy flow due to a plane electromagnetic wave propagating along z-direction in a dielectric medium is given by $\hat{z}\frac{k}{\omega\mu}E_0^2\cos^2(kz - \omega t),$ where $\mathbf{k}$ and ω are the propagation vector and angular frequency, E_0 is electric field amplitude, μ is the relative permeability of the medium.	TC	10	2014
14.	Derive the equation that represents Poynting's theorem. What is its physical significance?	TC	20	2015
15.	A radio station transmits electromagnetic waves isotropically with an average power of 200 kW. Determine the average magnitude of the maximum electric field at a distance of 5 km from it.	AN	15	2015
16.	A plane electromagnetic wave propagating along $+\hat{z}$ direction is incident normally on the boundary at $z = 0$ between medium $A(z < 0)$ and medium $B(z > 0)$. Determine the reflection coefficient and transmission coefficient for the wave.	TC	20	2015
17.	In free space, the electric field of electromagnetic wave is given as $\vec{E}(x, t) = 120\cos(\omega t - kx)\hat{y}$ V/m. Find the average power crossing a circular area of radius one metre in the yz-plane.	AN	10	2016
18.	Write down Maxwell's equations for linear dielectrics and deduce the equation of continuity.	TC	10	2016
19.	State and prove Poynting's theorem.	TC	20	2016

20.	Show that the displacement current between the plates of a parallel-plate capacitor is equal to the conduction current across the conductor.	TC	10	2016
21.	Write down the electromagnetic wave equations in non-conducting dielectric medium. Hence show that the velocity of wave propagation is given by $v = \sqrt{\frac{1}{\varepsilon\mu}}$, where the symbols have their usual meanings.	TC	10	2017
22.	Write down the physical significance of Maxwell's equations and explain the concept of displacement current by using a proper example.	TC	10	2017
23.	Define a plane electromagnetic wave. A plane polarized wave is incident on the interface between two dielectric media. Obtain expressions for the amplitudes of the reflected and transmitted waves when the incident wave is polarized with its electric field B vector perpendicular to the plane of incidence. Discuss the phase relationships of the reflected and transmitted waves with respect to the incident wave.	TC	20	2018
24.	Write down Maxwell's equations in integral form. Explain the significance of each of these equations.	TC	5	2018
25.	A parallel plate capacitor has plate area $= 4.0\ \mathrm{cm}^2$ and plate separation $=$ $2.0\ \mathrm{mm}$. An a.c. voltage $\mathrm{V} = 20\sin(5 \times 10^3 \mathrm{t})$ volts is applied across the plates. If the dielectric constant of the medium between the plates is $\varepsilon_r =$ 2.0, calculate the displacement current.	AN	5	2018
26.	Find the values of **E** and **H** on the surface of a wire carrying a current. By computing the Poynting vector, show that it represents a flow of energy into the wire.	TC	20	2019
27.	For the electric field given by $\mathrm{E} = \mathrm{E}_0 \mathrm{e}^{\mathrm{i}\omega \mathrm{t}}$, show that the conduction current is in phase with the electric field, while the displacement current leads the electric field by $\frac{\pi}{2}$ radians. Also, show that the displacement current in a good conductor is negligible compared to the conduction current at any frequency lower than the optical frequencies ($\mathrm{f} < 10^{15}\ \mathrm{Hz}$).	TC	10	2020
28.	For free space show that electromagnetic (EM) wave is transverse in nature. Show that for free space, the total outward flux of EM energy through surface S bounding volume V is equal to the rate of loss of EM energy from the volume V. A laser beam of $2\ \mathrm{mm}$ diameter has average power of $20\mathrm{GW}$. Calculate the peak values of electric and magnetic fields in the laser beam.	AN	20	2020
29.	Write Maxwell's equations in free space in both differential and integral forms. Obtain wave equations and show that electromagnetic waves can travel in free space with a speed of light. Can one get the wave equations from the integral form of the Maxwell's equations?	TC	15	2020
30.	A region $1, \mathrm{z} < 0$, has a dielectric material with $\varepsilon_\mathrm{r} = 3.2$ and a region 2 , $\mathrm{z} >$ 0 has a dielectric material with $\varepsilon_\mathrm{r} = 2.0$. Let the displacement vector in the region 1 be, $\vec{D}_1 = -30a_x + 50a_y + 70a_z \mathrm{nC}m^{-2}$. Assume the interface	AN	20	2021

	charge density is zero. Find in the region 2, the $\vec{\mathrm{D}}_2$ and $\vec{\mathrm{P}}_2$, where $\vec{\mathrm{P}}_2$ is the electric polarization vector in the region 2.			
31.	Calculate the skin depth of electromagnetic waves of $1\mathrm{MHz}$ incident on a good conductor having $\sigma = 5.8 \times 10^7 \mathrm{Sm}^{-1}$. Assume that inside the conductor $\mu = \mu_0 = 4\pi \times 10^{-7} \mathrm{Hm}^{-1}$.	AN	10	2021
32.	Write down Maxwell's equations in a non-conducting medium with constant permeability and susceptibility ($\rho = j = 0$). Show that $\vec{E}$ and $\vec{B}$ each satisfies the wave equation, and find an expression for the wave velocity. Write down the plane wave solutions for $\vec{E}$ and $\vec{B}$, and show how $\vec{E}$ and $\vec{B}$ are related.	TC	15	2022
33.	A metal guitar string with a length of $70\ \mathrm{cm}$ vibrates at its fundamental frequency of $246.94\ \mathrm{Hz}$ in a uniform magnetic field of $10\ \mathrm{T}$ oriented perpendicular to the plane of vibration of the string. Assume a sinusoidal form for the amplitude of the vibrational mode, and a maximum displacement of $3\ \mathrm{mm}$ at the centre of the string. What is the maximum e.m.f. generated across the length of the guitar string, and at what point in time in the string's motion does that occur? What would be the e.m.f. if the same guitar string vibrates at its second harmonic frequency? Briefly explain.	AN	20	2022
34.	A current sheet having $\vec{K} = 9.0a_y\ \mathrm{A\ m}^{-1}$ is located at $z = 0$. The interface is between the region $1, z < 0, \mu_{r_1} = 4$, and region $2, z > 0, \mu_{r_2} = 3$. Given that $\vec{H}_2 = 14.5a_x + 8.0a_z\ \mathrm{A\ m}^{-1}$. Find $\vec{H}_1$ and $\vec{B}_1$.	AN	15	2022

Black Body Radiation

35.	In deriving the Rayleigh-Jeans law, we count the number of modes dn corresponding to a wave number k for a photon gas in a cubical box. Consider a cubical container of volume V containing such gas in equilibrium. Calculate the differential number of allowable normal modes of frequency ω.	TC	20	2011
36.	State and explain Stefan-Boltzmann Law. Show that $\log\ P = \log\ K + 4\log\ R$, where P is the power emitted by black body and R is the resistance of the black body, K is a constant.	TC	10	2014
37.	Two spheres A and B having same temperature T are kept in the surroundings of temperature T_0. Consider $\mathrm{T} > \mathrm{T}_0$. The spheres are made of same material but have different, radii r_A and r_B. Using Stefan - Boltzmann distribution, determine which of these will lose heat by radiation faster.	TC	10	2015
38.	Using Planck's radiation law, deduce Wien's displacement law. How does this law enable one to estimate the surface temperature of the Sun or a star?	TC	15	2015

39.	What are the characteristic features of Rayleigh scattering? A very thin monochromatic beam of light is incident on a particle. Suggest a simple experimental method to ascertain whether the scattering by the particle is of Rayleigh type.	TC	20	2015
40.	The spectral energy curve of the moon shows maxima at $470\ \mathrm{nm}$ and $14\mu\mathrm{m}$. What inference can you draw from this data? Also calculate the energy density and radiation pressure in both cases. Given, Wien's constant $b = 2.892 \times 10^{-3}\ \mathrm{m\ K}$, Stefan's constant $\sigma = 5.67 \times 10^{-8}\ \mathrm{J\ m^{-2}\ s^{-1}\ K^{-4}}$ and speed of light $c = 3 \times 10^{8}\ \mathrm{m\ s^{-1}}$.	AN	10	2016
41.	Briefly explain Planck's law of blackbody radiation. Show that Planck's law reduces to Wien's law and Rayleigh-Jeans law at lower and higher wavelength limits respectively.	TC	20	2016
42.	Write down Stefan-Boltzmann law of radiation and derive it from Planck's law of radiation. An aluminium foil of relative emittance 0.1 is placed between two concentric spheres (assumed perfectly black) at temperatures $300\ \mathrm{K}$ and $200\ \mathrm{K}$ respectively. Find the temperature of the foil once the steady state is reached.	AN	15	2017
43.	What are the limitations of Rayleigh-Jeans law in explaining the spectrum of radiations from a blackbody? Explain how these limitations were overcome in Planck's radiation law. Deduce Wien's displacement law from Planck's radiation law.	TC	20	2018
44.	Discuss in brief the ultraviolet catastrophe. How did Planck solve this problem?	TC	10	2019
45.	Briefly outline the theory of scattering of electromagnetic radiation by a bound electron and hence derive the conditions for Rayleigh scattering. How can you explain the blue of the sky?	TC	20	2019
46.	A historic failure of Classical Physics is its inability to describe the electromagnetic radiation emitted from a black body. Consider a simple model for an ideal black body consisting of a cubic cavity of side L with a small hole on one side. Assuming the classical equipartition of energy, derive an expression for the average energy per unit volume and unit frequency range. In what way does this result deviate from actual observation? What is this law called? 	TC	20	2020

Repeat the calculations now using quantum idea to obtain an expression that properly accounts for the observed spectral distribution.

Find the temperature dependence of the total power emitted from the hole.

47. The spectral composition of solar radiation is similar to that of a black body radiator whose maximum emission corresponds to the wavelength $0.48\mu\mathrm{m}$. Find the mass lost by the Sun every second due to radiation. Evaluate the time interval during which the mass of the Sun reduces by 1 per cent. AN 20 2021

Given: Stefan Boltzmann constant $= 5.669 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$, radius of the Sun $= 6.957 \times 10^{8}\ \mathrm{m}$, surface temperature of the Sun $= 5772\ \mathrm{K}$ and mass of the Sun is $1.9885 \times 10^{30}\ \mathrm{kg}$.

THERMAL AND STATISTICAL PHYSICS

An UPSC CSE Physics Optional PYQ Repository

OCTOBER 3, 2023

ABHI PHYSICS

physicsupsc.com

Syllabus

(a) Thermodynamics:

- ✓ Laws of thermodynamics, reversible and irreversible processes, entropy; Isothermal, adiabatic, isobaric, isochoric processes and entropy changes;
- ✓ Otto and Diesel engines, Gibbs' phase rule and chemical potential; vander Waals equation of state of a real gas, critical constants;
- ✓ Maxwell-Boltzman distribution of molecular velocities, transport phenomena, equi-partition, and virial theorems;
- ✓ Dulong-Pet it, Einstein, and Debye's theories of specific heat of solids;
- ✓ Maxwell relations and applications;
- ✓ Clausius- Clapeyron equation; Adiabatic de-magnetisation, Joule-Kelvin effect and liquefaction of gases.

(b) Statistical Physics:

- ✓ Macro and micro states, statistical distributions,
- ✓ Maxwell-Boltzmann, Bose-Einstein, and Fermi-Dirac distributions,
- ✓ applications to specific heat of gases and black body radiation;
- ✓ Concept of negative temperatures.

Thermodynamics

1.	1kmol of an ideal gas is compressed isothermally at 400 K from 100kPa to 1000kPa in a piston and cylinder arrangement. Calculate the entropy change of the gas, the entropy change of the surroundings and the total entropy change resulting from the process if the process is mechanically reversible and the surroundings consist of a heat reservoir at 400 K.	AN	10	2010
2.	Calculate the change in pressure for a change in freezing point of water equal to $-0.91°\text{C}$. Given, the increase of specific volume when 1gm of water freezes into ice is 0.091cc/gm and latent heat of fusion of ice is 80cal/gm.	AN	10	2010
3.	Consider one mole of an ideal gas whose pressure changes with volume as $P = \alpha V$, where α is a constant. If it is expanded such that its volume increases m times, find the change in internal energy, work done by the gas and heat capacity of the gas.	TC	25	2010
4.	Derive an expression for the thermal efficiency of a reversible heat engine operating on the Diesel cycle with an ideal gas of constant heat capacity as the working medium.	TC	25	2010
5.	What led van der Waals to modify the ideal gas equation? Using the concepts of critical temperature T_c, pressure P_c and volume V_c, show that the critical constant for a real gas is $8/3$	TC	5 15	2011
6.	Find out the expressions for van der Waals constants a and b.	TC	20	2011
7.	Calculate the values of van der Waals constants a and b for oxygen with $T_c = 154.2\text{ K}, P_c = 49.7$ atmosphere and $R = 80\text{ cm}^3$ atmosphere /K.	AN	20	2011
8.	Show that the Helmholtz free energy of a system never increases in any isothermal-isochoric transformation.	TC	12	2012
9.	Establish the relation $\left(\frac{\partial T}{\partial V}\right)_P = -\left(\frac{\partial P}{\partial S}\right)_T$ and then derive $\left(\frac{\partial C_P}{\partial P}\right)_T = -T\left(\frac{\partial^2 V}{\partial T^2}\right)_P$. Hence show that the heat capacity C_P of an ideal gas is independent of pressure P.	TC	20	2012
10.	The Einstein theory of specific heat of solids gives the expression $C_V = \dfrac{3Nk_\text{B}x^2e^x}{(e^x-1)^2}$ where $x = \frac{\theta_E}{T}$ with θ_E as the Einstein temperature.	TC	15 10	2012

(i) Mention Einstein's assumptions in deriving it. Also obtain low- and high-temperature limiting expressions for it.
(ii) Give schematic plot of $\frac{C_V}{3Nk_B}$ versus $\frac{T}{\theta_E}$ and comment on the validity of expressions in (i) in comparison with experiments.

11. A thermally insulated ideal gas is compressed quasi-statically from an initial state with volume V_0 and pressure P_0 to a final state of volume V_f and pressure P_f. Show that the work done on the gas in the process is given by — TC 10 2013

$$W = \frac{C_V}{R}(P_f V_f - P_o V_o)$$

where C_v and R having standard meanings.

12. In a tungsten filament lamp, thermionic emission takes place at 1.2×10^3 K. Calculate the ratio of spontaneous emission to stimulated emission for non-degenerate energy levels. Interpret your result physically. — AN 10 2013
Take $\lambda = 550$ nm, $k_B = 1.38 \times 10^{-2}$ JK^{-1}, $h = 6.67 \times 10^{-34}$Js and $c = 3 \times 10^8$ ms^{-1}.

13. The vapour pressure, in mm of Hg, of a substance in solid state is given by the relation $\ln\ p = 23.03 - \frac{3754}{T}$, where T is in Kelvin. The vapour pressure, in mm of Hg, of the substance in liquid state is given by the relation $\ln\ p = 19.49 - \frac{3063}{T}$. Calculate (i) the coordinates of the triple point, and (ii) the latent heat of vaporisation at the triple point. Take Gas constant $R = 8.314$ J mol^{-1} K^{-1}. — AN 15 2013

14. In Leh, temperature of ice on a cold winter night is measured as -20°C. Calculate the change in entropy when 1 kg of ice is converted into steam at 100°C. Given specific heat capacity of ice is 500 cal kg^{-1} K^{-1}, latent heat of ice is 3.36×10^5 J kg^{-1}, latent heat of steam is 2.26×10^6 J kg^{-1} and J = 4.2 Jcal^{-1}. — AN 15 2013

15. Define Enthalpy and show that it remains constant in a throttling process. — TC 10 2014

16. In deriving radiation laws, we consider a cubical container of volume V containing a photon gas in equilibrium. Calculate the differential number of allowed normal modes of frequency ω. — TC 10 2014

17. One kg of water at 20°C is converted into ice at -10°C at constant pressure. Heat capacity of water is 4,200 J/kg. K and that of ice is 2,100 J/kg.K. Heat of fusion of ice at 0°C is 335×10^3 J/kg. Calculate the total change in entropy of the system. — AN 15 2014

18. Consider a system of free gas particles having f degrees of freedom. Use equipartition theorem to establish the relation TC 15 2014

$$f = \frac{2}{\left(\frac{C_p}{C_V} - 1\right)},$$

where C_p and C_V are molar specific heats at constant pressure and constant volume respectively. Obtain the values of $\frac{C_p}{C_V}$ for diatomic and triatomic gases.

19. Explain the four thermodynamic relations of Maxwell. Using the same, obtain the Clausius-Clapeyron equation TC 15 2014

$$\frac{\mathrm{dP}}{\mathrm{dT}} = \frac{\mathrm{L}}{\mathrm{T}(\mathrm{V}_2 - \mathrm{V}_1)}$$

20. A Van der Waals gas undergoes Joule-Kelvin expansion with a pressure drop of 50 atm. If its initial temperature is $300^\circ\mathrm{K}$, determine its final temperature. (Given Van der Waals constant $\mathrm{a} = 0.136\ \mathrm{Pa\ m^6\ mol^{-1}}$, $\mathrm{b} = 36.5 \times 10^{-6}\ \mathrm{m^3\ mol^{-1}}, \mathrm{C_p} = 30\ \mathrm{J\ K^{-1}\ mol^{-1}}, \mathrm{R} = 8.3\ \mathrm{J\ K^{-1}\ mol^{-1}}$) AN 10 2015

21. The vapour pressure of an organic substance is 50×10^3 Pa at $40^\circ\mathrm{C}$. Its normal boiling point is $80^\circ\mathrm{C}$. If the substance in vapour phase can be treated like an ideal gas, find the latent heat of vaporization of the substance. AN 15 2015

22. For a Van der Waals gas, write down the equation of state. Determine the coefficient of critical expansion β. TC 15 2015

23. m gram of water at temperature T_1 is isobarically and adiabatically mixed with an equal mass of water at temperature T_2. Show that the change in entropy is given by $\Delta S = 2mC_p \ln\left(\frac{T_{\mathrm{av}}}{T_{\mathrm{geo}}}\right)$, where $T_{\mathrm{av}} = \frac{T_1+T_2}{2}$ and $T_{\mathrm{geo}} = \sqrt{T_1 T_2}$. TC 10 2016

24. Write down van der Waals' equation of state for n moles of a gas and calculate the temperature at which 5 moles of the gas at 5 atm pressure will occupy a volume of 20 litres. AN 10 2016
Given, $R = 8.31 \times 10^7$ erg $\mathrm{mol^{-1}\ K^{-1}}$, $a = 1.34 \times 10^{12}$ dyne $\mathrm{cm^4\ mol^{-2}}$, $b = 31.2\ \mathrm{cm^3\ mol^{-1}}$ and 1 atm $= 1.013 \times 10^6$ dyne $\mathrm{cm^{-2}}$.

25. A student is working in a physics laboratory, which is at temperature $27^\circ\mathrm{C}$, on a sonometer to study formation of stationary waves. The cross-sectional area of the sonometer wire is $0.85 \times 10^{-6}\ \mathrm{m^2}$ and a tension of $20\ \mathrm{N}$ is applied on it. If the rigid supports are $1.2\ \mathrm{m}$ apart and the temperature of the wire drops by $7^\circ\mathrm{C}$, calculate the (i) final tension and (ii) fundamental frequency of vibration of the wire. Take, coefficient of linear AN 10 2016

	expansion and isothermal Young's modulus as $1.5 \times 10^{-5}\ \mathrm{K}^{-1}$ and $2.0 \times 10^{11}\ \mathrm{N\ m}^{-2}$ respectively.			
26.	What do you understand by the term 'phase transition'? Using Clausius-Clapeyron equation, show that for first-order phase transitions, vapour pressure decreases exponentially with temperature. You can assume that the vapour behaves like an ideal gas and latent heat remains constant with temperature.	TC	15	2016
27.	1 litre of hydrogen at $127^{\circ}\mathrm{C}$ and 10^6 dynes $/\mathrm{cm}^2$ pressure expands isothermally until its volume is doubled and then expands adiabatically until its volume is redoubled. Calculate the resulting pressure. $(\gamma = 1.42)$	AN	10	2017
28.	Derive Clausius-Clapeyron equation. How does it explain the effect of pressure on melting point of solids and boiling point of liquids?	TC	10	2017
29.	Calculate the critical temperature for helium, given the values for critical constants, $a = 6.15 \times 10^{-5}, b = 9.95 \times 10^{-4}$, where the unit of pressure is atm and the sample is kept at NTP.	AN	10	2017
30.	A reversible engine converts $1/6$ of the heat input into work. When the temperature of the sink is reduced by $62\ ^{\circ}\mathrm{C}$, its efficiency is doubled. Find the temperatures of source and sink.	AN	10	2017
31.	One mole of a gas obeys the following equation of state: $\left(P + \frac{a}{v^2}\right)(v - b) = RT,$ where v is the molar volume and, a and b are constants. Show that internal energy of the gas increases as the volume increases, with the temperature remaining constant.	TC	10	2018
32.	At $4^{\circ}\mathrm{C}$ temperature, the density of water is found to be maximum. Prove that heat capacity at the constant pressure (c_p) is equal to the heat capacity at constant volume $(\mathrm{c_v})$ for water at $4^{\circ}\mathrm{C}$.	TC	10	2018
33.	If the temperature variation of heat capacity is known, how do you calculate the change of entropy during an isochoric process? According to Debye's theory of specific heat of a solid, the molar heat capacity of diamond crystal at constant volume varies with temperature (T) as follows: $c_v = \frac{12}{5}\pi^4 R \left(\frac{T}{\Theta}\right)^3$ where R is the molar gas constant $= 8.315\ \mathrm{J/mol\ K}$ and $\Theta = 2230\ \mathrm{K}$ for diamond.	AN	20	2018

Calculate the change in entropy of diamond of 0.36 g mass when it is heated at constant volume from 0 K to 300 K.

34. The pressure on 100 g of solid copper is increased quasi-statically and isothermally at $0°\mathrm{C}$ from 0 to 0.5×10^8 Pa. Assuming the density and isothermal compressibility to remain at constant values of $8.96\ \mathrm{g/cm^3}$ and $7.16 \times 10^{-12}\ \mathrm{Pa^{-1}}$, respectively, calculate the work done. Comment on the sign and magnitude of work. AN 15 2018

35. What are the conditions for the change in temperature of a van der Waals gas passing through a porous plug? Prove that the ideal gas passing through the porous plug does not show any change in temperature. TC 10 2019

36. What is Gibbs' phase rule? Find the values of degrees of freedom when AN 15 2019
(i) only the liquid CO_2 is in equilibrium with the gaseous CO_2.
(ii) water is in the vapour-liquid saturation region.
(iii) water is in a single-phase region.
(iv) water is at the triple point.

37. Einstein's molar specific heat capacity of a solid is given by AN 20 2019

$$C_V = 3R\left(\frac{\theta_E}{T}\right)^2 \frac{e^{\theta_E/T}}{(e^{\theta_E/T}-1)^2},$$
$$\text{where } \theta_E = \frac{\hbar\omega}{k_B}$$

Obtain the expressions for the cases:
(i) when $T \gg \theta_E$
(ii) when $T \ll \theta_E$
What is the discrepancy of Einstein model to explain the variation of specific heat capacities of solids with the temperature?

The molar specific heat capacity of a solid at constant volume is $2.77\ \mathrm{JK^{-1}}$ at 36.8 K. Determine the Debye temperature of the solid.

38. What is Carnot's theorem? Prove that Carnot's reversible engine is the most efficient one and no other engine can be more efficient than Carnot's engine. TC 15 2019

39. If the partition function for a perfect gas is given by AN 15 2019

$$Z = \frac{V}{h^3}(2\pi mkT)^{3/2}$$

calculate (i) average kinetic energy per molecule and (ii) specific heat of the gas.

40. Explain the effect of pressure on the melting and boiling points of a substance using Clapeyron's latent heat equation. AN 15 2019

Calculate under what pressure, water will boil at $120^{\circ}\mathrm{C}$, if the change in specific volume when 1 gram of water is converted into steam is $1676\ \mathrm{cm}^3$. Latent heat of steam $= 540\mathrm{cal/g}, 1$ atmospheric pressure $=$ 10^6 dynes $/\mathrm{cm}^2$.

41. State the first law of thermodynamics for a diffusively interacting system. The temperature of $10\ \mathrm{g}$ of air is raised by $2^{\circ}\mathrm{C}$ at constant volume. Calculate the increase in its internal energy. Given: $\mathrm{C_v} = 0.172\ \mathrm{cal\ g^{-1}\ {}^{\circ}C^{-1}}$ — AN 10 2020

42. (i) The energy level of a quantum harmonic oscillator with frequency v is given by — AN 10 2020

$$\mathrm{E_n} = \left(\mathrm{n} + \frac{1}{2}\right)\mathrm{h}v, \text{ where } \mathrm{n} = 0,1,2, \ldots$$

Calculate its partition function.
(ii) Calculate the partition function of a two level system.

43. Discuss the principle of adiabatic demagnetization process to achieve low temperatures. Determine the fall in temperature produced by adiabatic demagnetization of a paramagnetic material at initial temperature of $3\ \mathrm{K}$ when the magnetic field is switched off from 10,000 oersted to zero. Given: heat capacity at constant magnetic field $= 0.2\ \mathrm{J\ g^{-1}\ K^{-1}}$ and Curie constant per gram mole per $\mathrm{cm}^3 = 0 \cdot 042\ \mathrm{erg\ K^{-1}\ g^{-1}\ Oe^{-2}}$. — AN 15 2020

44. Obtain the Clausius-Clapeyron equation. Using this equation, show that for the phase boundary of the liquid and vapour phases, $\mathrm{p} - \mathrm{T}$ relation can be written as $p = p_0 e^{-L/kT}$. Here it has been assumed that the latent heat L is independent of temperature, that vapour is treated as an ideal gas and that $\mathrm{V_{vapour}} = \mathrm{V} \gg \mathrm{V_{liquid}}$ and that $\mathrm{p} \rightarrow \mathrm{p_0}$ as $\mathbf{T} \rightarrow \infty$. — TC 15 2020

45. Calculate the critical constants for $\mathrm{CO_2}$ for which the Van der Waals constants are given by $a = 0.0072$ and $b = 0.002$. Also calculate the Boyle's temperature of $\mathrm{CO_2}$. The unit of pressure is atmosphere and the unit of volume is that of a gm-mole of the gas at NTP. — AN 10 2021

46. Explain the characteristics of the following thermodynamic processes for a perfect gas: — TC 20 2021
(i) Isothermal process
(ii) Adiabatic process
(iii) Isobaric process
(iv) Isochoric process
Obtain the expression for the work done by the gas during the above processes.

47.	The melting point of tin is $232^{\circ}\mathrm{C}$, its latent heat of fusion is $14\ \mathrm{cal/g}$ and the specific heat of solid and molten tin are 0.055 and $0.064\ \mathrm{cal/g}\ ^{\circ}\mathrm{C}$ respectively. Calculate the change in entropy when $1.0\mathrm{gm}$ of tin is heated from $100^{\circ}\mathrm{C}$ to $300^{\circ}\mathrm{C}$.	AN	10	2021
48.	Calculate the efficiency of an engine having compression ratio 13.8 and expansion ratio 6 and working on diesel cycle. Given $\gamma = 1 \cdot 4$.	AN	5	2021
49.	Assume that the Earth's atmosphere is pure nitrogen in thermodynamic equilibrium at a temperature of $300\ \mathrm{K}$. Calculate the height above sea level at which the density of the atmosphere is one-half its sea level value. (Molecular weight of N_2 is $28\mathrm{gm/mole}$)	AN	10	2022
50.	A body of constant heat capacity C_p and a temperature T_i is put into contact with a reservoir at temperature T_f. Equilibrium between the body and the reservoir is established at constant pressure. Determine the total entropy change and prove that it is positive for either sign of $\left[(T_f - T_i)/T_f\right]$. Consider $\frac{\lvert T_f - T_i \rvert}{T_f} < 1$.	TC	10	2022
51.	One mole of gas obeys van der Waals equation of state. If its molar internal energy is given by $u = cT - a/V$ (in which V is the molar volume, a is one of the constants in the equation of state and c is a constant), calculate the molar heat capacities C_v and C_p.	TC	10	2022
52.	A compressor designed to compress air is used instead to compress helium. It is found that the compressor overheats. Explain this effect, assuming that the compression is approximately adiabatic and the starting pressure is same for both the gases. $\left[\gamma_{\mathrm{He}} = \frac{5}{3}, \gamma_{\mathrm{Air}} = \frac{7}{5}\right]$	TC	10	2022
53.	A gas of interacting atoms has an equation of state and heat capacity at constant volume given by the expressions $p(T,V) = aT^{1/2} + bT^3 + cV^{-2}$ $C_v(T,V) = dT^{1/2} + eT^2V + fT^{1/2}$ where a through f are constants which are independent of T and V. Find the differential of the internal energy $dU(T,V)$ in terms of dT and dV.	TC	15	2022
54.	A thermally insulated cylinder, closed at both ends, is fitted with a frictionless heat-conducting piston which divides the cylinder in two parts. Initially, the piston is clamped in the centre, with one litre of air at $200\ \mathrm{K}$ and $2\ \mathrm{atm}$ pressure on one side and one litre of air at $300\ \mathrm{K}$ and $1\ \mathrm{atm}$ pressure on the other side. The piston is released and the system reaches equilibrium in pressure and temperature, with the piston at a new position. Compute the final pressure and temperature.	AN	15	2022

55. In a partially conducting medium, $\varepsilon_r = 18.5, \mu_r = 800$ and $\sigma = 1\ \mathrm{S\ m^{-1}}$. Find α, β, η and the velocity u, for a frequency of 10^9 Hz. Determine $\vec{H}(z,t)$. Given, $\vec{E}(\mathrm{z},t) = 50e^{-\alpha z}\ cos(\omega t\ -\ \beta a_z)a_y\ \mathrm{V\ m^{-1}}$ AN 20 2022

Statistical Physics

56. Calculate the number of different arrangements of 10 indistinguishable particles in 15 cells of equal a priori probability, considering that one cell contains only one particle. AN 10 2010

57. Consider the following statement: TC 10 2010
"The Fermi energy of a given material is the energy of that quantum state which has the probability equal to $\frac{1}{2}$ of being occupied by the conduction electrons."
Is the above statement correct? Give reasons for your answer.

58. Write down the expressions for Bose-Einstein and Fermi-Dirac distribution functions, and show how Fermi-Dirac distribution leads to the explanation of Pauli's exclusion principle. TC 10 2011

59. Consider non-equilibrium situation for a system in which the population inversion has been achieved. Explain that such a system can be treated as if it has negative absolute temperature. TC 12 2012

60. Show that both Fermi-Dirac and Bose-Einstein distributions reduce under certain condition in a form which gives the total number of particles as TC 15 2012
15

$$N = A\int_0^\infty \sqrt{\varepsilon}e^{-\beta\varepsilon}d\varepsilon$$

where A is a constant and $\beta = 1/k_\mathrm{B}T$. Further show that this expression is just the same as that obtained from the Maxwellian speed distribution.

61. The coefficient of viscosity of helium at 27°C is $2 \times 10^{-5}\ \mathrm{kg\ m^{-1}\ s^{-1}}$. Calculate (i) the average speed and (ii) the diameter of a helium molecule, if it is assumed that the gas obeys Maxwell-Boltzmann distribution. Given Boltzmann constant $\mathrm{k_B} = 1.38 \times 10^{-23}\ \mathrm{J\ K^{-1}}$ and mass of helium atom $= 6.67 \times 10^{-27}\ \mathrm{kg}$. AN 10 2013

62. N particles obeying Classical Statistics are distributed among three states having energies $\varepsilon_1 = 0, \varepsilon_2 = \mathrm{k_B T}$ and $\varepsilon_3 = 2\mathrm{k_B T}$, where $\mathrm{k_B}$ is Boltzmann constant. If the total equilibrium energy of the system is $1000\mathrm{k_B T}$, calculate the value of N. AN 10 2013

63. Show that both Fermi-Dirac and Bose-Einstein distribution functions at an energy E are given by: TC 10 2014

$$\mathrm{f(E)} \simeq \exp\,[(\mu - \mathrm{E})/\mathrm{k_B T}],$$

where $\mathrm{f(E)}$ is much smaller than unity, μ and $\mathrm{k_B T}$ are the chemical potential and thermal energy of the atom.

64. Using Maxwell-Boltzmann distribution law prove that there cannot be any negative absolute temperature. TC 10 2014 2017

65. The molecules of a gas obey Maxwell-Boltzmann distribution. Calculate the fraction of molecules of the gas within 1 % of the most probable speed at STP. Interpret your result. TC 10 2016

66. Consider a system of N particles and a phase space consisting of only two states with energies 0 and $\varepsilon(> 0)$. Obtain the expressions for the partition function and the internal energy of the system, if it obeys M-B statistics. TC 20 2016

67. The viscosity in a liquid arises due to friction between adjacent layers. What causes viscosity in a gas? Explain. TC 5 2016

68. The molecules of a gas obeying Maxwell-Boltzmann distribution move with an average speed of $450\ \mathrm{m\ s^{-1}}$. If the coefficient of viscosity of the gas η is $16.6 \times 10^{-6}\mathrm{Nsm^{-2}}$, density of the gas ρ is $1.25\ \mathrm{kg\ m^{-3}}$ and number density is $2.7 \times 10^{25}\ \mathrm{m^{-3}}$, calculate the mean free path and diameter of the gas molecules. AN 10 2016

69. Write and explain the Maxwell-Boltzmann distribution. Using this distribution, find the expressions for the most probable speed, mean speed and root-mean-square speed. TC 15 2017

70. Explain Bose-Einstein distribution and obtain the same from the grand canonical ensemble. TC 15 2017

71. A system having two energy levels, $-\frac{1}{2}\Delta$ and $+\frac{1}{2}\Delta$ with $\Delta = 10\mathrm{meV}$ is populated by 1000 particles at a low temperature close to $100\ \mathrm{K}$. Obtain the average energy per particle using classical distribution law. AN 15 2018

72. Schematically, show the variation of density of states, $D(\varepsilon)$ and distribution function, $\mathrm{f}(\varepsilon, \mathrm{T})$, of particles in a non-relativistic Fermi gas at high temperatures. TC 20 2018
At a temperature T, an electron occupies a state with energy $100\mathrm{meV}$ above the Fermi energy (ε_F) with the probability of 1%. Find the temperature T.

73. A gas has only two particles, a and b. With the help of a diagram, show that how these two particles can be arranged in the three quantum series TC 10 2019

	1, 2, 3 using (i) Maxwell-Boltzmann, (ii) Fermi-Dirac, and (iii) Bose-Einstein statistics.			
74.	Starting from Maxwell-Boltzmann distribution for a free particle in 3-dimension, obtain the expression for root mean square (rms) speed of a particle. Calculate the rms speed of nitrogen (N_2) molecule at room temperature $(27^\circ\mathrm{C})$.	AN	20	2020
75.	Eight indistinguishable balls are to be arranged in six distinguishable boxes. Calculate the total number of ways in which the above can be done.	TC	10	2021
76.	Write the expression for the Fermi-Dirac distribution. Plot the Fermi-Dirac distribution at $\mathrm{T} = 0$ and for $\mathrm{T}_1 > \mathrm{T}_2 > 0$. Now from the plot propose two alternative definitions of the Fermi level.	TC	15	2021
77.	Calculate the probability of an electron occupying an energy level $0.02\mathrm{eV}$ above the Fermi level at $\mathrm{T} = 300\ \mathrm{K}$	AN	5	2021
78.	What do you understand by negative temperature? Write and explain various restrictions on a system for the concept of negative temperature to be meaningful.	TC	15	2022

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QUANTUM MECHANICS

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ABHI PHYSICS

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Syllabus

- ✓ Wave-particle duality; Schrodinger equation and expectation values;

- ✓ Uncertainty principle; Solutions of the one-dimensional Schroedinger equation for a free particle (Gaussian wave-packet), particle in a box, particle in a finite well, linear harmonic oscillator;

- ✓ Reflection and transmission by a step potential and by a rectangular barrier;

- ✓ Particle in a three-dimensional box, density of states, free electron theory of metals;

- ✓ Angular momentum; Hydrogen atom; Spin half particles, properties of Pauli spin matrices

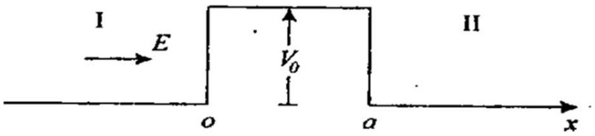
Foundations of Quantum Mechanics

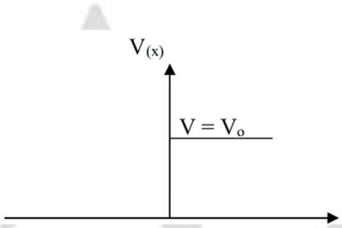
1.	Derive Bohr's angular momentum quantization condition in Bohr's atomic model from the concept of de Broglie waves.	TC	10	2010
2.	Calculate the wavelength of de Broglie waves associated with electrons accelerated through a potential difference of 200 Volts.	AN	10	2011
3.	Estimate the size of the hydrogen atom and the ground state energy from the uncertainly principle.	AN	10	2011
4.	Calculate $(\Delta x)^2$, where $\Delta x = x - \langle x \rangle$	TC	15	2011
5.	Use the uncertainly principle to estimate the ground state energy of a linear harmonic oscillator	TC	12	2012
6.	In a series of experiments on the determination of the mass of a certain elementary particle, the results showed a variation of $\pm 20\ \mathrm{m_e}$, where $\mathrm{m_e}$ is the electron mass. Estimate the lifetime of the particle.	AN	10	2013
7.	Find the de Broglie wave length of i. a neutron ii. an electron moving with kinetic energy of $500eV$ $(1eV = 1.602 \times 10^{-19}J)$	AN	10	2014
8.	The mean life of Lambda ($\Lambda°$) particle is 2.6×10^{-10} s. What will be the uncertainty in the determination of its mass in eV ?	AN	10	2014
9.	Find the energy, momentum and wavelength of photon emitted by a hydrogen atom making a direct transition from an excited state with $\mathrm{n} = 10$ to the ground state. Also find the recoil speed of the hydrogen atom in this process.	AN	10	2016
10.	An electron is confined to move between two rigid walls separated by 10^{-9} m. Compute the de Broglie wavelengths representing the first three allowed energy states of the electron and the corresponding energies.	AN	10	2016
11.	A typical atomic radius is about $5 \times 10^{-15}\ m$ and the energy of β-particle emitted from a nucleus is at most of the order of 1 MeV. Prove on the basis of uncertainty principle that the electrons are not present in nuclei.	TC	10	2016

12.	Using uncertainty principle, calculate the size and energy of the ground state hydrogen atom.	TC	10	2016
13.	A beam 4.0 keV electrons from a source is incident on a target 50.0 cm away. Find the radius of the electron beam spot due to Heisenberg's uncertainty principle.	AN	10	2017
14.	Estimate the de Broglie wavelength of the electron orbiting in the first excited state of the hydrogen atom.	TC	10	2017
15.	Show that the mass and linear momentum of a quantum mechanical particle can be given by $m = h/(\lambda v)$ and p = h/λ, respectively, where h, λ and v are Planck's constant, wavelength, and velocity of the particle, respectively. Comment on the wave-particle duality from these relations.	TC	10	2019
16.	State and express mathematically the three uncertainty principles of Heisenberg. Highlight the physical significance of these principles in the development of Quantum Mechanics.	TC	10	2019
17.	For a free quantum mechanical particle under the influence of a one-dimensional potential, show that the energy is quantized in discrete fashion. How do these energy values differ from those of a linear harmonic oscillator?	TC	10	2019
18.	Using the uncertainty principle $\Delta x \Delta p \geq \hbar/2$, estimate the ground state energy of a harmonic oscillator.	TC	15	2020
19.	A blue lamp emits light of mean wavelength of 4500 Å. The rating of the lamp is 150 W and its 8% of the energy appears as light. How many photons are emitted per second by the lamp?	AN	10	2020
20.	Consider a Hermitian operator A with property $A^3 = 1$. Show that $A = 1$.	TC	15	2020
21.	Find the uncertainty in the momentum of a particle when its position is determined within 0.02 cm. Find also the uncertainty in the velocity of an electron and α-particle respectively when they are located within $15 \times 10^{-8} cm$.	AN	15	2021
22.	A particle of rest mass m_0 has a kinetic energy K, show that its de Broglie wavelength is given by $\lambda = \dfrac{hc}{\sqrt{[K(K + 2m_0c^2)]}}$	AN	15	2021

	Hence calculate the wavelength of an electron of kinetic energy 2MeV. What will be the value of λ if $K << m_0c^2$?			
23.	What is de Broglie concept of matter wave? Evaluate de Broglie wavelength of Helium that is accelerated through 300V. (Given mass of proton = mass of neutron = $1.67\ \times 10^{-27}kg$)	AN	10	2022

Schrodinger's Wave equation and applications

24.	Obtain an expression for the probability current for the plane wave $\psi(x,t) = \exp\,[i(kx - \omega t)]$. Interpret your result.	TC	10	2010 2015
25.	A system is described by the Hamiltonian operator, $H = -\frac{d^2}{dx^2} + x^2$ Show that the function A x exp $\left(-\frac{x^2}{2}\right)$ is an eigen function of H. Determine the eigen values of H.	TC	30	2010
26.	 A stream of particles of mass M and energy E is directed from left to a one-dimensional potential barrier as shown in the above figure. Set up the time-independent Schrodinger equation and obtain an expression for transmission probability from region I to II. How this phenomenon helps in the understanding of α-decay of nuclei?	TC	30	2010
27.	Solve the Schrödinger equation for a particle of mass m in an infinite rectangular well defined by $V_{(x)} = \begin{cases} 0 & ;\ 0 \leq x \leq L \\ \infty & ;\ x < 0, x > L \end{cases}$ Obtain the normalized eigen functions and the corresponding eigen values.	TC	25	2011
28.	Normalize the wave function $\psi(x) = e^{-\lvert x \rvert}\sin\,\alpha x$	TC	10	2011
29.	Consider the one-dimensional wavefunction $\psi(x) = Axe^{-kx}, (0 \leqslant x \leqslant \infty; k > 0)$	TC	4	2012

	i. Calculate A so that $\psi(x)$ is normalized.		8	
	ii. Using Schrödinger's equation find the potential $V(x)$ and energy E for which $\psi(x)$ is an eigenfunction. (Assume that as $x \rightarrow \infty, V(x) \rightarrow 0$).			
30.	a) Solve the radial part of the time-independent Schrödinger equation for a hydrogen atom. Obtain an expression for the energy eigenvalues.	TC	45 15	2012
	b) What is the degree of degeneracy of the energy eigenvalues? What happens if the spin of the electron is taken into account?			
31.		TC	30 10 10	2013
	a) Consider a beam of particles incident on a one-dimensional step function potential with energy $\mathrm{E} > \mathrm{V_o}$ as shown in the above figure. Solve the Schrödinger equation and obtain expressions for the reflection and transmission coefficients.			
	b) What are the limits of the reflection coefficient for $\mathrm{E} \rightarrow \mathrm{V_o}$ and $\mathrm{E} \rightarrow \infty$?			
	c) Discuss the cases $0 < \mathrm{E} < \mathrm{V_o}$ and E < 0.			
32.	Obtain the time-dependent Schrödinger equation for a particle. Hence deduce the time independent Schrödinger equation.	TC	20	2014
33.	Solve the Schrödinger equation for a particle of mass m confined in one dimensional potential well of the form: $V_{(x)} = \begin{cases} 0 & ;\ 0 \le x \le L \\ \infty & ;\ x < 0, x > L \end{cases}$ Obtain the discrete energy values and the normalized eigen functions.	TC	20	2014
34.	An electron is moving in a one-dimensional box of infinite height and width 1 Å. Find the minimum energy of electron.	TC	10	2014
35.	Normalized wave function of a particle is given: $\psi(\mathrm{x}) = \mathrm{N}\,\mathrm{exp}\left(\frac{-x^2}{2a^2} + \mathrm{ikx}\right)$. Find the expectation value of position.	AN	10	2015

36.	Write the time independent Schrödinger equation for a bouncing ball.	TC	10	2015
37.	Solve the Schrodinger equation for a step potential and calculate the transmission and reflection coefficient for the case when the kinetic energy of the particle E_0 is greater than the potential energy V (i.e., $\mathrm{E}_0 > \mathrm{V}$).	TC	20	2016
38.	Calculate the lowest energy of an electron confined to move in a 1-dimensional potential well of width 10nm.	AN	10	2017
39.	Using Schrodinger equation, obtain the eigenfunctions and eigenvalues of energy for a 1- dimensional harmonic oscillator. Sketch the profiles of eigenfunctions for first three energy states.	TC	20	2017
40.	Calculate the probability of transmission of an electron of 1.0 eV energy through a potential barrier of 4.0 eV and 0.1 nm width.	AN	10	2017
41.	The wave function of a particle is given as $\psi(x) = \frac{1}{\sqrt{a}} e^{-\lvert x \rvert/a}$. Find the probability of locating the particle in the range $-a \leq x \leq a$.	AN	10	2018
42.	Calculate the zero-point energy of a system consisting of a mass of 10^{-3} kg connected to a fixed point by a spring which is stretched by 10^{-2} m by a force of 10^{-1} N. The system is constrained to move only in one direction.	AN	10	2018
43.	The general wave function of harmonic oscillator (one-dimensional) are of the form $$u_n(x) = \sum_{k=0}^{\infty} a_k y^k e^{-y^2/2}$$ With $y = \sqrt{\frac{m\omega}{\mathrm{h}}}x$, and coefficients a_k are determined by recurrence relations $$a_{k+2} = \frac{2(k-n)}{(k+1)(k+2)} a_k$$ Corresponding energy levels are $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$. Discuss the parity of these wave functions. What happens, if the potential for $x \leq 0$ is infinite (half harmonic oscillator)?	TC	10	2018
44.	Which of the following functions is/are acceptable solution(s) of the Schrodinger equation? i. $\psi(x) = Ae^{-ikx} + Be^{ikx}$ ii. $\psi(x) = Ae^{-kx} + Be^{kx}$	AN	15	2018

iii. $\psi(x) = A\sin 3\,kx + B\cos 5\,kx$

iv. $\psi(x) = A\sin 3\,kx + B\sin 5\,kx$

v. $\psi(x) = A\tan\,kx$

Explain your answer.

45. A beam of particles of energy 9 eV is incident on a potential step 8 eV high from the left. What percentage of particles will reflect back? AN 15 2018

46. Write down the Hamiltonian operator for a linear harmonic oscillator. Show that the energy eigenvalue of the same can be given by $E_n = \left(n + \frac{1}{2}\right)\hbar\omega_0$ at energy state n with ω_0 being the natural frequency of vibration of the linear oscillator. Prove that $n = 0$ energy state has a wave function of typical Gaussian form. TC 15 2019

47. Estimate the size of hydrogen atom and the ground state energy from the uncertainty principle. TC 15 2019

48. Prove that Bohr hydrogen atom approaches classical conditions, when n becomes very large and small quantum jumps are involved. TC 10 2020

49. Find the probability current density for the wave function TC 10 2020

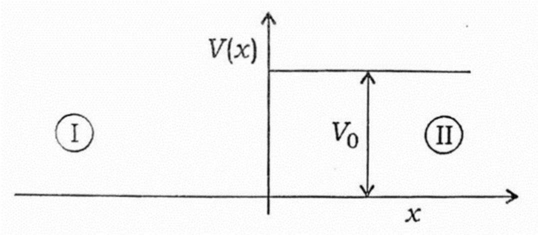
$$\Psi(\chi, \mathrm{t}) = \left[\mathrm{A}\mathrm{e}^{\mathrm{ipx}/\hbar} + \mathrm{B}\mathrm{e}^{-ipx/\hbar}\right]\mathrm{e}^{-ip^2t/2m\hbar}$$

Interpret the result physically.

50. A particle is described by the wave function $\Psi(\mathrm{x}) = \left(\frac{\pi}{2}\right)^{-1/4} e^{-ax^2/2}$. Calculate Δx and Δp for the particle, and verify the uncertainty relation $\Delta\mathrm{x}\Delta\mathrm{p} = \frac{\hbar}{2}$. AN 15 2020

51. Write the wave functions for a particle on both sides of a step potential, for $E > V_0$: TC 10 2020

$$V(x) = \begin{cases} V_0, x > 0 \\ 0,\ \ x < 0 \end{cases}$$



Interpret the results physically.

52. Normalised wave function of hydrogen atom for $1s$ state is AN 10 2021

$$\psi_{100} = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}, \text{ where } a_0 = \frac{\hbar^2}{me^2}$$

being the Bohr radius. Calculate the expectation value of potential energy in this state.

53. A beam of $12\mathrm{eV}$ electron is incident on a potential barrier of height $25\mathrm{eV}$ and width $0.05\ \mathrm{nm}$. Calculate the transmission coefficient. AN 10 2021

54. A particle is moving in a one-dimensional box of width 50Å and infinite height. Calculate the probability of finding the particle within an interval of 15Å at the centres of the box when it is in its state of least energy. AN 15 2021

55. Calculate the probability of finding a simple harmonic oscillator within the classical limits if the oscillator is in its normal state. Also show that if the oscillator is in its normal state, then the probability of finding the particle outside the classical limits is approximately 16%. TC 15 2021

56. An electron in a one-dimensional infinite potential well, defined by $V(x) = 0$ for $-a \leqslant x \leqslant a$ and $V(x) = \infty$ otherwise, goes from $n = 4$ to $n = 2$ level and emits photon of frequency $3.43 \times 10^{14}\ \mathrm{Hz}$. Calculate the width of the well. (Assume Plank's constant $h = 6.626 \times 10^{-34}$ J.S. and mass of electron $m = 9.11 \times 10^{-31}\ \mathrm{kg}$) AN 10 2022

57. Set up the Schrodinger's wave equation for one dimensional potential barrier and obtain the probability of tunnelling. TC 20 2022

Angular momentum, spin etc

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58. Show that the Pauli spin matrices satisfy the following: TC 10 2010

$$\begin{gathered}\sigma_x^2 = \sigma_y^2 = \sigma_z^2 = 1 \\ \sigma_x\sigma_y = -\sigma_y\sigma_x = i\sigma_z \\ \sigma_y\sigma_z = -\sigma_z\sigma_y = i\sigma_x \\ \sigma_z\sigma_x = -\sigma_x\sigma_z = i\sigma_y\end{gathered}$$

59. The normalized wave function for the electron in hydrogen atom for the ground state is $\psi(r) = (\pi a_0^3)^{-1/2} \exp\left(-\frac{r}{a_0}\right)$ Where a_0 is the radius of the first Bohr orbit. Show that the most probable position of the electron is a_0 TC 10 2010

60.	Let $\vec{\sigma}$ be the vector operator with component equal to Pauli's spin matrices $\sigma_x, \sigma_y, \sigma_z$. If $\vec{a}\ and\ \vec{b}$ are vectors in 3D space, prove the identity $(\vec{\sigma}\cdot\vec{a})(\vec{\sigma}\cdot\vec{b}) = \vec{a}\cdot\vec{b} + i\vec{\sigma}\cdot(\vec{a}\times\vec{b})$	TC	10	2011
61.	The normalized wave function for the electron in the ground state of the hydrogen atom is given by $\psi(r) = \frac{1}{(\pi a_0^3)^{1/2}} e^{(-r/a_0)}$ where a_0 is the radius of the first Bohr orbit. Calculate $\langle r\rangle$ and $\left\langle\frac{1}{r}\right\rangle$.	TC	20	2011
62.	Given that $\sigma_x, \sigma_y, \sigma_z$ are Pauli spin operators, prove the following relationships: i. $\sin(\sigma_x\varphi) = \sigma_x \sin\varphi$ ii. $\cos(\sigma_z\varphi) = \cos\ \varphi$	TC	6 6	2012
63.	Using the definition $\vec{L} = \vec{r}\times\vec{p}$ of the orbital angular momentum operator, evaluate $[L_x, L_y]$	TC	10	2013
64.	The normalized wave function for the electron in the ground state of the hydrogen atom is given by $\psi(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-\frac{r}{a_0}}$, where a_0 is the radius of the first Bohr orbit. Calculate the probability of finding the electron within a distance r_0 of the proton in the ground state	TC	10	2013
65.	If $\hat{x}$ and $\hat{p}$ are the position and momentum operators, prove the commutation relation $[\hat{p}^2, \hat{x}] = -2i\hbar p$	TC	10	2014
66.	Write down Pauli spin matrices. Express J_x, J_y and J_z in terms of Pauli spin matrices.	TC	10	2014
67.	Using the commutation relations $[x, p_x] = [y, p_y] = [z, p_z] = i\hbar$ deduce the commutation relation between the components of angular momentum operator L. $[L_x, L_y] = i\hbar L_z$ $[L_y, L_z] = i\hbar L_x$ and $[L_z, L_x] = i\hbar L_y$.	TC	20	2014
68.	Solve the Schrödinger equation for a particle in a three-dimensional rectangular potential barrier. Explain the terms degenerate and non-degenerate states in this context.	TC	30	2015

69.	A particle trapped in an infinitely deep square well of width a has a wave function $\psi = \left(\frac{2}{\pi}\right)^{1/2} \sin\left(\frac{\pi x}{a}\right)$. The walls are suddenly separated by infinite distance. Find the probability of the particle having momentum between p and $\mathrm{p} + \mathrm{dp}$.	AN	10	2015
70.	Write down the matrix representation of the three Pauli matrices $\sigma_\mathrm{x}, \sigma_\mathrm{y}$ and σ_z. Prove that these matrices satisfy the following identities : (i) $\left[\sigma_\mathrm{x}, \sigma_\mathrm{y}\right] = 2\ \mathrm{i}\ \sigma_\mathrm{z}$ (ii) $[\sigma^2 \cdot \sigma_\mathrm{x}] = 0$ (iii) $(\vec{\sigma} \cdot \vec{A})(\vec{\sigma} \cdot \vec{B}) = \vec{A} \cdot \vec{B} + i\vec{\sigma} \cdot (\vec{A} \times \vec{B})$ if $\vec{\mathrm{A}}$ and $\vec{\mathrm{B}}$ commute with Pauli matrices.	TC	8 4 4 4	2016
71.	Calculate the density of states for an electron moving freely inside a metal with the help of quantum mechanical Schrodinger's equation for free particle in a box.	TC	10	2016
72.	Evaluate the most probable distance of the electron from nucleus of a hydrogen atom in its $2p$ state. What is the probability of finding the electron at this distance?	AN	20	2017
73.	Explain why the square of the angular momentum (L^2) and only one of the components $\left(\mathrm{L}_\mathrm{x}, \mathrm{L}_\mathrm{y}, \mathrm{L}_\mathrm{z}\right)$ of L are regarded as constants of motion.	TC	15	2017
74.	The ground state wave function for hydrogen atom is $$\psi(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$$ where a_0 is the Bohr radius. Sketch the wave function and the probability density as a function of the separation distance r. Calculate the probability that the electron in the ground state is found beyond the Bohr radius.	TC	20	2018
75.	Prove the following identities: (i) $\left[\hat{p}_x, \hat{L}_y\right] = i\hbar\hat{p}_z$ (ii) $e^{i(\hat{\sigma}\cdot\hat{n})\theta} = \mathrm{co\,s}\,\theta + i(\hat{\sigma} \cdot \hat{n})\mathrm{si\,n}\,\theta$	TC	7 8	2018
76.	Show that for free electron gas, the density of states in three dimensions (3D) varies as $E^{1/2}$, and this dependence changes to E^0	TC	15	2018

	for $2D$ (quantum well), $E^{-1/2}$ for 1D (quantum wire) and δ function for OD (quantum dot).			
77.	How do you define density of states? Show that the density of states with wave vector less than $\vec{k}$ in a three-dimensional cubic box of volume V can be given by $D(\omega) = \frac{V}{2\pi^2} k^2 \left(\frac{dk}{d\omega}\right)$ in the frequency spectrum between ω and $\omega + d\omega$. Here, assume that the number of modes per unit range of k is $L/(2\pi)$, L being the length of each side of the cubic box.	TC	20	2019
78.	Define Pauli spin matrices σ_x, σ_y and σ_z. Using these definitions, prove the following: (i) $\sigma_x^2 = \sigma_y^2 = \sigma_z^2 = 1$ (ii) $\sigma_x\sigma_y = i\sigma_z; \sigma_z\sigma_x = i\sigma_y; \sigma_y\sigma_z = i\sigma_x$	TC	15	2019
79.	Define angular momentum of a particle and find out the three components of the angular momentum operator $\hat{L}$ in Cartesian coordinates. Show that $\hat{L}^2 = -\hbar^2 \left[r^2\nabla^2 - \frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right)\right]$ Prove that the operator $\hat{L}^2$ can also be expressed as $\hat{L}^2 = -\hbar^2 \left[\frac{1}{\sin\,\theta}\frac{\partial}{\partial\theta}\left(\sin\,\theta \frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\,\theta}\frac{\partial^2}{\partial\phi^2}\right]$ in spherical polar coordinates (r, θ, ϕ).	TC	20	2019
80.	If the z-component of an electron spin is $+\frac{\hbar}{2}$, what is the probability that its component along a direction z' (forming an angle θ with z-axis) is $\frac{\hbar}{2}$ or $-\frac{\hbar}{2}$? What is the average value of spin along z' ?	TC	20	2020
81.	Prove the commutation relation for the angular momentum: $[L^2, L_z] = 0$ Also show that $(\vec{L} \times \vec{L}) = i\hbar\vec{L}$.	TC	15	2020
82.	Using Pauli spin matrices prove that,	TC	8	2021
	(i) $\sigma_x\sigma_y + \sigma_y\sigma_x = 0; \sigma_y\sigma_z + \sigma_z\sigma_y = 0; \sigma_x\sigma_z + \sigma_z\sigma_x = 0$		6	
	(ii) $\sigma_+\sigma_- = 2(1 + \sigma_z)$		6	
	(iii) $\sigma_\alpha + \sigma_\beta = i\sigma_\gamma$ where $\alpha \neq \beta \neq \gamma$			
83.	Show that $E_n = \langle V \rangle$ in the stationary states of the hydrogen atom.	TC	15	2022

84. Obtain the normalized eigenvectors of σ_x and σ_y matrices. TC 15 2022

ATOMIC & MOLECULAR PHYSICS

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ABHI PHYSICS
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Syllabus

- ✓ Stern-Gerlach experiment, electron spin, fine structure of hydrogen atom;
- ✓ Elementary ideas about Lamb shift and its significance, Zeeman effect;
- ✓ L-S coupling, J-J coupling; Spectroscopic notation of atomic states;
- ✓ Elementary theory of rotational, vibrational and electronic spectra of diatomic molecules, Frank Condon principle and applications;
- ✓ Raman effect and molecular structure; Laser Raman spectroscopy;
- ✓ Importance of neutral hydrogen atom, molecular hydrogen and molecular hydrogen ion in astronomy;
- ✓ Fluorescence and Phosphorescence; Elementary theory and applications of NMR and EPR;

Atomic Physics

1.	What is Zeeman effect? How it can be understood on quantum mechanical basis? Obtain an expression for Zeeman splitting of atomic energy levels in a magnetic field B. Explain the magnetic splitting of sodium D-lines.	TC	30	2010
2.	What is spin-orbit interaction? Calculate the energy shift due to spin-orbit interaction term in H-like system. Discuss the significance of this shift in relation to the fine structure of hydrogen spectral lines	TC	30	2010
3.	Discuss the fine structure of hydrogen atom spectrum. Draw the compound doublet spectrum arising as a result of transitions between ^2P and ^2D levels.	TC	10	2011
4.	What do you mean by 'term symbols'? Obtain term symbols for the following sets of values of S and L : (i) $S = \frac{1}{2}, L = 2$ (ii) $S = 1, L = 1$ (iii) $S = \frac{3}{2}, L = 1$	TC	10	2011
5.	Show that $^2S_{\frac{1}{2}}$, $^2P_{\frac{1}{2}}$ and $^2P_{\frac{3}{2}}$ levels of sodium spectrum are split in the ratio of $3:1:2$ due to anomalous Zeeman effect.	AN	30	2011
6.	What is Lamb shift?	TC	5	2011
7.	Sodium doublets are produced by transitions $3^2p_{1/2} \rightarrow 3^2s_{1/2}(D_1)$ and $3^2p_{3/2} \rightarrow 3^2s_{1/2}(D_2)$. Calculate the Landé g-factors for these levels.	AN	12	2012
8.	What will happen to the energy level of an unpaired electron when subjected to an external magnetic field? Find the condition for electron spin resonance.	TC	20	2012
9.	How are electrons distributed in the various sub-shells for $n = 3$? Give the quantum numbers for the electrons in the second shell.	AN	10	2013

10.	The term symbol for atomic states are quoted as $^3P_2$ and $^2D_{5/2}$. What are the values of L, S and J ?	AN	10	2013
11.	Discuss the fine structure of sodium D line. Draw D_1 and D_2 lines due to the transitions between ^2P and ^2S levels.	TC	25	2013
12.	Show that the velocity of electron in the first orbit of hydrogen atom is $\left(\frac{1}{137}\right) C$ where C is the velocity of light. (Given electronic charge $=$ 1.602×10^{-1} C Planck Constant 6.63×10^{-34} J. s, permittivity $=$ $8.85 \times 10^{-1}\ \mathrm{C}^2\ \mathrm{N}^{-1}\ \mathrm{m}^{-2}$)	AN	15	2013
13.	A sample of a certain element is placed in a magnetic field of flux density 0.3 tesla. How far apart is the Zeeman component of a spectral line of wavelength 4500Å ? Given : $\mathrm{e/m} = 1.76 \times 10^{11} \mathrm{c/kg}, \mathrm{c} = 3.0 \times 10^8\ \mathrm{ms}^{-1}$.	AN	10	2014
14.	Obtain an expression for the normal Zeeman shift. Illustrate the Zeeman splitting of spectral lines of H atom and the allowed transitions for the $l\ =\ 1$ and $l\ =\ 2$ states.	AN	20	2014
15.	Explain how the nuclear spin I depends on the mass number A and atomic number Z of atoms.	TC	10	2014
16.	The energy levels of a hydrogen atom are given by $E_n = \left(\frac{-1}{n^2}\right) R_{yd}$ where $1\mathrm{R_{yd}} = \mathrm{hcR}$. Show that $\mathrm{R} = 1.097 \times 10^7\ \mathrm{m}^{-1}$.	AN	5	2015
17.	If K, L and M energy levels of platinum are approximately 78, 12 and 3 keV, respectively, below the vacuum level, calculate the wavelengths of K_α and K_β lines.	AN	10	2015
18.	What is Zeeman effect? How can it be understood on the basis of quantum mechanics?	TC	25	2015
19.	Obtain Zeeman splitting for sodium D-lines.	TC	15	2015
20.	Find the magnetic moment of an atom in $3\mathrm{P}_2$ state, assuming that LS coupling holds for this case.	AN	10	2015

21.	In the Stern-Gerlach experiment using Ag atoms, the oven temperature is $1000\ \mathrm{K}, l \approx 25\ \mathrm{cm}$ and $\frac{\partial \mathrm{B_z}}{\partial \mathrm{z}} \simeq 10^{+3}\mathrm{Tesla/m}$. Calculate the separation of the two components.	AN	10	2016
22.	Describe Stern-Gerlach experiment. Discuss how it has explained space quantization and electron spin. Find the value of angle between the spin angular momentum $\vec{\mathrm{S}}$ and its z-component of an electron moving along the external magnetic field $\vec{\mathrm{B}}$.	TC	10 10 10	2016
23.	The series limit wavelength of Balmer series in hydrogen spectrum is experimentally found to be 3646Å. Find the wavelength of the first line of this series.	AN	10	2016
24.	Compute the allowed spectral terms for two non-equivalent p-electrons on the basis of Pauli's exclusion principle.	TC	10	2016
25.	Explain in detail L-S coupling and j-j coupling schemes.	TC	10	2016
26.	What is Lamb shift? What is its significance in determining the fine structure of H_α Balmer line in hydrogen atom?	TC	10	2016
27.	Show that the lines in the absorption spectra corresponding to the rotational transitions from two adjacent energy levels of a medium sized molecule at room temperature have comparable intensities.	TC	10	2017
28.	State Franck-Condon principle. Define Franck-Condon factors. Using schematic diagram, explain the decay of excited states leading to the phenomena of fluorescence and phosphorescence.	TC	20	2017
29.	Calculate the radius of electron orbit for Li^{++}in ground state.	AN	10	2018
30.	Describe the importance of L-S and J-J coupling in atomic spectroscopy. What are experimental evidences of their existence?	TC	20	2018
31.	What is Zeeman effect? Discuss the factors on which Larmor frequency is dependent.	TC	15	2018

32.	Discuss the fine structure of hydrogen spectrum. How is it of importance in the astronomical observations?	TC	15	2018
33.	Describe normal and anomalous Zeeman effect. Explain how it lifts the degeneracy in hydrogen atom	TC	20	2019
34.	What is Lamb shift? Discuss its significance in determining the fine structure of H_α Balmer line in hydrogen atom.	TC	15	2019
35.	Define mathematically the Bohr radius of a hydrogen atom and show that the binding energy at state n of this atom can be given by $E_n = -\frac{1}{2}\frac{Ze^2}{(a/Z)}\frac{1}{4n^2\pi\varepsilon_0}$ where Z is the atomic number of H atom. Calculate the numerical values of a and E_1 of H atom.	TC	15	2019
36.	Determine the normal Zeeman splitting of the cadmium red line of 6438 Å, when the atoms are places in a magnetic field of 0.009 T.	AN	15	2020
37.	Explain how the magnetic moments of atoms, the space quantization of angular momentum and the spin of electron are measured using Stern – Gerlach experiment.	TC	15	2020
38.	Find the minimum magnetic field needed for the Zeeman effect to be observed in a spectral line of $400\ \mathrm{nm}$ wavelength when a spectrometer whose resolution is $0.010\ \mathrm{nm}$ is used. Write the answer in the nearest high integer.	AN	10	2021
39.	Calculate the Larmor precessional frequency for a magnetic induction field of $0.5\ \mathrm{T}$. Hence calculate the splitting in wave numbers of a spectral line due to normal Zeeman effect for the same field.	AN	10	2021
40.	Explain spin-orbit coupling. Discuss the splitting of spectral lines of H-atom due to spin-orbit coupling.	TC	15	2021
41.	The quantum numbers of two electrons in a two-valence electron atom are;	AN	7 8	2021

	$n_1 = 8 \quad l_1 = 4 \quad s_1 = \frac{1}{2}$ $n_2 = 7 \quad l_2 = 2 \quad s_2 = \frac{1}{2}$ (i) Assuming $L - S$ coupling, find the possible value of L and hence of J. (ii) Assuming $j - j$ coupling, find the possible values of J.			
42.	Describe normal and anomalous Zeeman effect. Explain how it lifts the degeneracy in hydrogen atom.	TC	20	2021
43.	Show that the Landé g-factor for pure orbital angular momentum and pure spin angular momentum are 1 and 2 respectively. Further, evaluate the g-factor for the state ${}^3P_1$.	TC	10	2022
44.	The raising $(J_+)$and lowering (J_-)operators are defined by $J_+ = J_x + iJ_y$ and $J_- = J_x - iJ_y$ respectively. Prove the following identities : (i) $[J_z, J_\pm] = \pm\hbar J_\pm$ (ii) $J_- J_+ = J^2 - J_z^2 - \hbar J_z$	TC	10	2022
45.	Show that for a given principal quantum number n, there are n^2 possible states of the atom.	TC	7	2022
46.	An atomic state is denoted by ${}^4D_{5/2}$. Find the values of L, S and J. For this state, what should be the minimum number of electrons involved ? Suggest a possible electronic configuration.	TC	8	2022
47.	What is the spin wave function (for $s = \frac{1}{2}$) if the spin component in the direction of unit vector η has a value of $\frac{1}{2}\hbar$?	TC	15	2022
48.	(i) Why does Stern-Gerlach experiment enjoy so much importance in atomic physics? (ii) Draw the schematic diagram of this experiment and comment on the shapes of the magnet pole pieces. (iii) Why was the atomic beam of silver used in this experiment?	TC	20	2022

49.	Calculate, giving necessary steps, the radio frequency at which nuclear magnetic resonance occurs in water kept in a uniform magnetic field of $2.4\ \mathrm{T}$. The magnetic moment of proton is $2.793\mu\mathrm{N}$.	AN	10	2010
50.	State and explain Franck-Condon principle. Discuss its applications in molecular spectroscopy.	TC	10	2010
51.	Discuss occurrence of rotational energy levels of a diatomic molecule and show that the pure rotation spectrum of such a molecule consists of a series of equally spaced lines separated by a constant wave number difference $2B$. Write down the selection rules.	TC	15	2010
52.	Is it possible to obtain pure rotational spectra of $\mathrm{H_2}, \mathrm{HF}, \mathrm{O_2}$ and NO molecules?	TC	5	2010
53.	In CO molecule $J = 0 \rightarrow J = 1$ line occurs at a frequency $1.153 \times 10^{11}\ \mathrm{Hz}$. Calculate the moment of inertia of CO molecule.	AN	10	2010
54.	What is Raman Effect? How does it differ from Rayleigh scattering? Explain Raman Effect on the basis of quantum mechanical theory. How is Raman Effect experimentally studied? What are the advantages of using laser sources in the study of Raman Effect?	TC	30	2010
55.	On the basis of three principal moments of inertia I_A, I_B and I_C each about X, Y and Z axes respectively, how can you classify molecules?	TC	30	2011
56.	Treating a diatomic molecule as a simple harmonic oscillator, obtain its vibrational energy levels.	TC	20	2011
57.	The observed vibrational frequency of the CO molecule is $6.42 \times 10^{13}\ \mathrm{Hz}$. What is the effective force constant of this molecule? (Mass of carbon atom $= 12u$ and mass of oxygen atom $= 16\mathbf{u}$, where u is atomic mass unit)	AN	10	2011
58.	Discuss pure rotational spectra of linear molecules.	TC	25	2011
59.	Why should rotational Raman spectrum show a separation of the first Raman line from the exciting line equal to $6B\ \mathrm{cm}^{-1}$, while the separation between successive lines equals to $4B\ \mathrm{cm}^{-1}$, where B is the rotational constant?	AN	12	2012

60.	Describe the vibration-rotation spectra of a polyatomic linear molecule. What are P, Q and R branches? Draw rotational energy levels of the vibrational stales $v = 0$ and $v = 1$ and perpendicular vibrational-rotational transitions of a linear polyatomic molecule.	AN	40	2012
61.	What do you understand by H-one (HI) interstellar clouds and their importance to understand the universe.	TC	10	2013
62.	With proper selection rules, construct the energy level diagram and allowed transitions for ESR spectrum of hydrogen atom.	TC	30	2013
63.	Why are Raman active vibrations and infrared vibrations in CO_2 molecule complementary to each other?	TC	10	2013
64.	In a Raman spectrum of a linear triatomic molecule, the first three lines are 4.86, 8.14 and $11.36\ \mathrm{cm}^{-1}$. Calculate the rotational constant, B and the moment of inertia of the molecule. (Given $h = 6.626 \times 10^{-27}\ \mathrm{J.s}, C = 3.0 \times 10^{10}\ \mathrm{cm/sec.}$)	AN	10	2013
65.	Given that the spacing between vibrational levels of CO molecules is $8.45 \times 10^{-2}\mathrm{eV}$ of energy. Find the force constant of the molecule.	AN	10	2014
66.	Discuss the vibrational spectra of a diatomic molecule treating it as an anharmonic oscillator.	TC	20	2014
67.	Obtain an expression for the resonance condition in NMR.	TC	10	2014
68.	Explain the relaxation processes in NMR spectroscopy.	TC	10	2014
69.	Two successive lines in the rotational emission spectrum of HCl molecule appear at wave numbers $83.5\ \mathrm{cm}^{-1}$ and $104.1\ \mathrm{cm}^{-1}$. Calculate the position of the next line appearing at the higher wave number.	AN	10	2015
70.	Establish that:	AN	5	2015

$$\begin{aligned} \mathrm{hc} &= 1240\ \text{eV.nm} \\ &= 1240 \mathrm{MeV.fm} \end{aligned}$$

71.	Hydrogen molecule is diatomic. Obtain the rotational energy levels of this molecule. Write down the selection rules. Obtain the smallest energy required to excite the lowest rotational mode.	TC	30	2015
72.	The observed vibrational frequency of CO molecule is $6.42 \times 10^{13}\ \mathrm{Hz}$. What is the effective force constant of the molecule?	AN	10	2015
73.	What is Raman effect? Describe briefly the chief characteristics of pure rotational spectra. The small rotational Raman displacement for HCl molecule is $41.6\ \mathrm{cm}^{-1}$. Find the internuclear distance between the atoms forming the molecule.	AN	10 10 10	2016
74.	Given the force constant of HCl molecule $= 516\mathrm{Nm}^{-1}$, determine the wave number of the fundamental mode of vibration of the molecule. How many transition lines one can expect in the vibration spectra of HCl molecule at room temperature?	AN	10	2017
75.	Explain Stokes and anti-Stokes Raman scattering with the help of energy level diagram. For a diatomic molecule, obtain expressions for transition energies of its Raman spectra with rotational fine structure and hence the wave numbers of the Stokes lines.	TC	20	2017
76.	Explain why lines in some Raman spectra are found to be plane polarized to different extents even though the exciting radiation is completely unpolarized?	TC	10	2017
77.	Explain the principle of Nuclear Magnetic Resonance (NMR) with the help of an energy level diagram. Give examples of nuclei which exhibit NMR. What major inferences can be drawn from an NMR spectra?	TC	20	2017
78.	In an NMR experiment, hydrogen atoms are subjected to a magnetic field of $5.0\ \mathrm{T}$. Determine the difference in energy $(\mathrm{kJ/mol})$ between two spin states of the nuclei of hydrogen atom and the frequency of radiation required for NMR.	AN	15	2017
79.	What is nuclear precession? How is it used in the principle of working of an NMR?	TC	10	2018

80.	Discuss the theory of rotational and vibrational spectra of diatomic molecules. What is the difference between fluorescence and phosphorescence?	TC	20	2018
81.	Why is population inversion in general not possible in a two-level laser system? Explain it.	TC	10	2019
82.	Why are Raman active vibrations and IR vibrations in $\mathrm{CO_2}$ molecule complementary to each other?	TC	10	2019
83.	What is Franck-Condon principle? Discuss the intensity distribution in the vibrational electronic spectra of a diatomic molecule on the basis of this principle.	TC	15	2019
84.	Write the principle of nuclear magnetic resonance (NMR). Explain the design and working of NMR, and write its important applications.	TC	20	2020
85.	Calculate the frequency of the first Bohr orbit of hydrogen atom.	TC	15	2020
86.	What is Zeeman effect? Explain Zeeman effect on the basis of classical electron theory.	TC	15	2020
87.	The potential energy of a diatomic molecule in terms of the interatomic spacing R is given by $$U(R) = \frac{-A}{R^2} + \frac{B}{R^{10}}$$ where $A = 1.44 \times 10^{-3}\ \mathrm{Jm^2}$ and $B = 2.19 \times 10^{-115}\ \mathrm{J\ m^{10}}$. Calculate the equilibrium spacing, R_e and the dissociation energy.	AN	20	2020
88.	The first line in the pure rotational spectrum of HCl appears at $21 \cdot 18\ \mathrm{cm^{-1}}$. Calculate bond length of the molecule. Given atomic masses of H and Cl are 1.008 and $35.45\mathrm{amu}$, respectively.	AN	10	2021
89.	In observing the Raman spectrum of a sample using 3637Å as the exciting line, one gets stoke line at 3980Å. Deduce the Raman shift in $\mathrm{m^{-1}}$ units. Compute the wavelength in Å for corresponding stokes and anti-stokes lines if the exciting line is 6465Å.	AN	20	2021

90.	Calculate the magnetic field strength required to observe the NMR spectrum of protons in benzene at $120\mathrm{MHz}$. [Given the value of nuclear g-factor g_N for protons is 5.585]	AN	10	2022
91.	Define Franck-Condon principle. How does it help in explaining the intensity distribution of vibrational-electronic spectra of diatomic molecules.	TC	15	2022
92.	(i) In a diatomic molecule when one constituent atom is replaced by one of its heavier isotopes, what change takes place in the rotational spectrum? (ii) Calculate the change in rotational constant B when hydrogen is replaced by deuterium in the hydrogen molecule. (iii) Draw the spectra of rigid and non-rigid rotors by using the schematic representation of the rotational energy levels and comment on it.	TC	20	2022
93.	(i) Briefly explain the effect of anharmonicity on the vibrational spectra of diatomic molecules. (ii) Calculate the average period of rotation of HCl molecule if it is in the $J = 3$ state. The internuclear distance and the moment of inertia of HCl are $0 \cdot 1274\ \mathrm{nm}$ and $0 \cdot 0264 \times 10^{-45}\ \mathrm{kg} \cdot \mathrm{m}^2$ respectively.	AN	15	2022

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Syllabus

- Basic nuclear properties-size, binding energy, angular momentum, parity, magnetic moment;
- Semi-empirical mass formula and applications, mass parabolas; Ground state of deuteron, magnetic moment and non-central forces;
- Meson theory of nuclear forces; Salient features of nuclear forces;
- Shell model of the nucleus - successes and limitations;
- Violation of parity in beta decay; Gamma decay and internal conversion; Elementary ideas about Mossbauer spectroscopy;
- Q-value of nuclear reactions; Nuclear fission and fusion, energy production in stars; Nuclear reactors.
- Classification of elementary particles and their interactions;
- Conservation laws; Quark structure of hadrons; Field quanta of electro weak and strong interactions;
- Elementary ideas about unification of forces; Physics of neutrinos.

Nuclear Physics

1.	Calculate the packing fraction and the binding energy per nucleon for $^{16}_{8}\mathrm{O}$ and $^{87}_{38}\mathrm{Sr}$ nuclei.	AN	10	2010
2.	A star converts all its hydrogen to helium, achieving 100% helium. It then converts the helium to carbon via the reaction $$^{4}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{12}_{6}\mathrm{C} + 7 \cdot 27\mathrm{MeV}$$ The mass of the star is 5.0×10^{32} kg and it generates energy at the rate of 5×10^{30} W. How long will it take to convert all Helium to carbon at this rate?	AN	10	2010
3.	What are magic numbers? Discuss shell structure of a nucleus. How is this model able to explain various properties of nuclei? Discuss the limitations of this model.	TC	25	2010
4.	Predict from the single particle shell model the shell configuration, ground state spin and parity for the following nuclei: $$^{13}_{7}\mathrm{N};\ ^{17}_{8}\mathrm{O}$$	TC	10	2010
5.	Explain parity violation in β-decay. Describe how parity violation was experimentally detected in the decay of $^{60}\mathrm{Co}$.	TC	25	2010
6.	Show that nucleus is a quantum system.	TC	10	2011
7.	Find the total kinetic energy of electron and antielectron neutrino emitted in beta decay of free neutron. (The neutron-proton mass difference is 1.30 MeV and mass of electron is 0.51 MeV)	AN	10	2011
8.	What is the importance of study of deuteron? Obtain the solution of Schrodinger equation for ground state of deuteron and show that deuteron is a loosely bound system.	TC	25	2011 2021
9.	What do you mean by non- central forces?	TC	5	2011
10.	What are chain reactions? What do you mean by critical size of the core in which chain reactions take place? What is critical mass?	TC	20	2011
11.	$^{235}\mathrm{U}$ yields two fragments of $\mathrm{A} = 95$ and $\mathrm{A} = 140$. Obtain the energy distribution of the fission products. Assume that the two fragments are ejected with equal and opposite momentum.	AN	10	2011
12.	Determine from the given data whether the following reactions are exothermic or endothermic: (i) $^{1}_{1}\mathrm{H} + {}^{3}_{1}\mathrm{H} \rightarrow {}^{2}_{1}\mathrm{H} + {}^{2}_{1}\mathrm{H}$ (ii) $^{12}_{6}\mathrm{C} + {}^{12}_{6}\mathrm{C} \rightarrow {}^{20}_{10}\mathrm{Ne} + {}^{4}_{2}\mathrm{He}$	AN	12	2012
13.	Calculate the recoil energy of $^{57}_{26}\mathrm{Fe}$ nucleus when it emits a gamma photon of energy $14\mathrm{keV}$.	AN	12	2012

14.	Write down the Bethe-Weizsäcker semiempirical mass formula for a nucleus. Explain the significance of each term occurring in it. Discuss the stability of a nucleus against β-decay. What is the effect of pairing term on stability?	TC	45	2012
15.	What are mirror nuclei? How does the charge independence of nuclear force emerges from this concept?	TC	15	2012
16.	Calculate the Q-value of the reaction: $^{9}_{4}\mathrm{Be}(^{4}_{2}\mathrm{He},\mathrm{n})^{12}_{6}\mathrm{C}$Given : Mass $(^{9}_{4}\mathrm{Be}) = 9 \cdot 01283\mathrm{u}$ Mass $(^{4}_{2}\mathrm{He}) = 4 \cdot 002603\mathrm{u}$ Mass $(^{12}_{6}\mathrm{C}) = 12 \cdot 000\mathrm{u}$ Mass $(\mathrm{n}) = 1.0086652\mathrm{u}$	AN	10	2013 2015
17.	Write down the nucleonic configuration of, $^{7}\mathrm{Li}$, $^{12}\mathrm{C}$, $^{17}\mathrm{O}$ and $^{27}\mathrm{Al}$ in the ground tate of the nuclear shell model and hence calculate the corresponding ground state angular momenta and parities. How do the observed ground state angular momenta and parities agree with those predicted on the basis of shell model?	AN	10	2013
18.	Explain parity violation in β-decay. Describe how parity violation was experimentally detected in the decay of $^{60}\mathrm{Co}$. Mention any other decay process in which the parity violation has been demonstrated.	TC	20	2013
19.	What is the role of neutrino in the weak interaction of radioactive nuclides? Explain the experimental detection of neutrino.	TC	15	2013
20.	Explain why stable light nuclei have equal number of protons and neutrons whereas heavy nuclei have excess of neutron.	TC	10	2014
21.	It is possible to estimate the nuclear radius from the study of alpha decay? Explain how.	TC	10	2014
22.	What are salient features of nuclear forces?	TC	10	2014
23.	Discuss Yukawa's theory of nuclear forces.	TC	10	2014
24.	How does liquid drop model explain fission?	TC	10	2014
25.	What are the limitations of shell model?	TC	10	2014
26.	What are the magic numbers in nuclei? List the experimental evidences indicating their existence.	TC	20	2014
27.	Explain why the deuteron has no excited state.	TC	10	2016
28.	State the basic assumption of single particle shell model. How do the centrifugal and spin-orbit terms remove the degeneracy of a three-dimensional spherical harmonic oscillator?	TC	10 15	2016

29.	Show that in the nuclear shell model, the level spacing between major oscillator shells is approximately $h\omega = 41\ A^{-1/3}\mathrm{MeV}$.	TC	15	2016
30.	Predict the spin and parity of ground states of the following nuclei on the basis of shell model: (i) ${}_8O^{15}$ (ii) ${}_8O^{16}$ (iii) ${}_{17}Cl^{38}$	AN	10	2016
31.	Estimate the order of nuclear radius of lead $(Z = 82)$ using the large angle (back) scattering of alpha particles of energy $10\mathrm{MeV}$ incident on a target (lead). [Given: $(4\pi\epsilon_0)^{-1} = 9 \times 10^9 \mathrm{Nm^2C^{-2}}$]	AN	10	2017
32.	Distinguish between charge independence and charge symmetry of nuclear force. Give one example of each of these.	TC	10	2017
33.	Describe briefly how parity violation in β-decay was experimentally observed? What do you understand by the statement, 'neutrinos are left-handed'?	TC	10	2017
34.	Write two properties of deuteron which support the existence of non-central tensor force.	TC	5	2017
35.	Given that the deuteron magnetic moment operator (in units of nuclear magneton) can be expressed as $\vec{\mu}_d = \mu_n\vec{\sigma}_n + \mu_p\overrightarrow{\sigma_p} + \frac{1}{2}\vec{l}$ where $\vec{l}$ is the relative angular momentum between neutron and proton, $\vec{\sigma}_n$ and $\vec{\sigma}_p$ are the Pauli spin operators and μ_n and μ_p are the respective magnetic moments. Find out the D-state probability of deutron wave function. [Given : $\mu_d = 0.857\mu_N, \mu_n = -1.913\mu_N$ and $\mu_p = 2.793\mu_N$; μ_N (nuclear magneton)]	AN	10	2017
36.	How does one explain the approximate constancy of average binding energy per nucleon (BE/A) of nuclei in the region $30 \leqslant A \leqslant 170$ in the plot of $\mathrm{BE/A}$ versus mass number A?	TC	5	2017
37.	Write the semi-empirical mass formula pointing out the role of volume term, surface energy term, coulomb and symmetry energy correction terms.	TC	15	2017
38.	Draw a schematic diagram of the single particle energy levels in a shell model including the effect of spin-orbit coupling. Show how it explains magic numbers in nuclei. Give two examples to show how this scheme predicts the spins and particles of odd A nuclei.	TC	20	2017
39.	Nuclear forces are mediated by exchange of π-mesons of rest mass 140 MeV. Estimate the range of nuclear forces.	AN	10	2018

40. The maximum energy of a positron (e^+) released in the decay of $_6\mathrm{C}^{13}$ atom into a $_7\mathrm{N}^{13}$ atom is $1.202\mathrm{MeV}$. If the mass of the $_6\mathrm{C}^{13}$ atom is $13.003354\mathrm{u}$, calculate the mass of the $_7\mathrm{N}^{13}$ atom. AN 10 2018

41. Assuming that the neutron-proton interaction has a square well form AC 10

$$\begin{aligned} V(r) &= -V_0 \qquad \text{for } r \le b \\ &= 0 \qquad\quad \text{for } r > b \end{aligned}$$

10

the ground state wave function of deuteron nucleus is given as

$$\begin{aligned} \psi(r) &= A\sin\, kr \text{ for } r \le b \\ &= Ce^{-\gamma r} \text{ for } r > b \end{aligned}$$

$$where\ k = \sqrt{\frac{M}{\hbar^2}(V_0 + W)}$$

$$\text{and } \gamma = \sqrt{\frac{MW}{\hbar^2}}.$$

Here M is the nucleon mass, W is the binding energy of deuteron and A and C are constants.

(i) Show that for a just bound state of deuteron

$$V_0 b^2 = \frac{\pi^2 n^2}{4M}$$

(ii) Explain why deuteron is a loosely bound extended structure.

42. A π⁻-meson at rest decays into a μ⁻-meson: AN 15 2018

$$\pi^- \to \mu^{\mp\bar{\nu}_\mu}$$

Calculate the kinetic energy of the μ⁻-meson emitted in the reaction.

43. Write the semi-empirical mass formula for nuclei and on its basis draw mass parabolas for odd and even isobars. What would be the most stable isobar in each case? TC 15 2018

44. Obtain an expression for the magnetic moment of a nucleus having one nucleon outside the closed core. Use this to calculate the magnetic moment of $_8\mathrm{O}^{17}$ nucleus. AN 20 2018

45. Stable light nuclei have equal number of protons and neutrons, whereas heavy nuclei have excess of neutrons. Explain why. TC 10 2019

46. Assuming equal masses for up (u) and down (d) quarks, find the ratio $(\mu_\mathrm{n}/\mu_\mathrm{p})$ of the magnetic moments of neutron and proton. TC 10 2019

47. Explain the various methods of finding the size of the nucleus. How will you determine the nuclear radius from the observation of beta rays resulting from nuclear transition when the initial and final nuclei are mirror nuclei? TC 5 15 2019

48.	Given that the single particle energy separation between $1d_{5/2}$ and $1d_{3/2}$ in ^{17}O is $5MeV$. Calculate the strength of spin-orbit interaction. It is observed that $1d_{5/2}$ level is lower than $1d_{3/2}$ level.	AN	15	2019
49.	Why is it not possible to detect the parity violation in weak interaction by observing only the beta decay rate? Justify your answer.	TC	15	2019
50.	Explain the origin of the nuclear magnetic moment. Deduce expression for the magnetic dipole moment with the help of the Schmidt single particle model.	TC	10	2019
51.	For a system consisting of one proton and one neutron (not necessarily a deuteron), write down the various possible states specifying clearly its isospin, spin and orbital quantum numbers.	TC	10	2019
52.	In a certain cyclotron, the maximum radius that the path of a deuteron may have before it is deflected out of the magnetic field is 20 cm. (i) Calculate the velocity of the deuteron at this radius. (ii) What is the energy of deuteron in MeV? Given, magnetic field $= 1500$ gauss and mass of deuteron $= 3.34 \times 10^{-27}$ kg.	AN	10	2020
53.	Calculate in terms of the nuclear magneton, μ_N, the magnetic dipole moment of $^3S_1$ state of deuteron. Given, $\mu_p = 2 \cdot 792847\mu_N$ and $\mu_n = -1.913042\mu_N$	AN	15	2020
54.	Explain the Mössbauer effect.	TC	10	2020
55.	Natural uranium found in the earth's crust contains the isotopes $^{235}_{92}U$ and $^{238}_{92}U$ in the atomic ratio $7 \cdot 3 \times 10^{-3}$ to 1. Assuming that at the time of formation these two isotopes were produced equally, estimate the time since formation. Given that the mean lives of both the isotopes are 1.03×10^9 years and 6.49×10^9 years, respectively.	AN	10	2020
56.	Write down the Weizsäcker mass formula for the nuclear binding energy. Give short justification for each term of the formula.	TC	20	2020
57.	List the main reactions in the *pp* chain leading from hydrogen to helium during stellar nucleosynthesis. Also mention the net effect of the reactions.	TC	10	2020
58.	What are chain reactions? What do you mean by critical size of the core in which chain reaction takes place?	TC	10	2021
59.	Show that in the nuclear shell model, the level spacing between major oscillator shells is approximately $\hbar\omega = 41A^{-1/3}MeV$.	TC	15	2021
60.	If the nuclear force is charge independent and a neutron and proton form a bound state then why is there no bound state for two neutrons? What information does this provide on the nucleon-nucleon force?	TC	10	2022

61.	Show that for a specific value (n, l), there exists a large degeneracy relative to the energy characterized by the quantum number (N). Find the shell closures and the magic numbers predicted by harmonic oscillator potential.	TC	15	2022
62.	Write down the Weizsäcker semi-emperical mass formula and explain each term. Explain why ${}^{238}_{92}\mathrm{U}$ nuclide is an α-emitter and not a β^{-}-emitter?	TC	10 10	2022
63.	A neutron and a proton can undergo radiative capture at rest: $$\mathrm{n} + \mathrm{p} \longrightarrow \mathrm{d} + \gamma$$ Find the energy of the photon emitted in this capture. Is the recoil of the deuteron important?	TC	15	2022

Particle Physics

64.	State the quantum numbers $\mathrm{I_z}, \mathrm{Y}$ and S for the *uds* quarks and antiquarks. Which combination of these leads to the formation of (i) proton and (ii) neutron?	TC	10	2010 2013 2015
65.	Explain conservation of Baryon Number. Comment on the stability of proton.	TC	10	2011
66.	Which of the following reactions are permitted or forbidden by various conservation laws? (i) $\mathrm{k}^{\mp}\mathrm{p} \rightarrow \Lambda^{\circ} + \pi^{\circ}$ (ii) $\mathrm{k}^{+} + \mathrm{n} \rightarrow \Sigma^{+} + \pi^{\circ}$ (iii) $\pi^{\mp}\mathrm{p} \rightarrow \mathrm{k}^{\mp}\Sigma^{+}$ (iv) $\pi^{+} + \mathrm{n} \rightarrow \mathrm{k}^{+} + \Lambda^{\circ}$ (v) $\pi^{\mp}\mathrm{p} \rightarrow \Lambda^{\circ} + \mathrm{k}^{\circ}$ (vi) $\pi^{\mp}\mathrm{p} \rightarrow \mathrm{k}^{+} + \Sigma^{-}$	TC	12	2012
67.	In the following reactions indicate with an explanation, whether they proceed by strong, electromagnetic or weak interaction or they are forbidden: (i) $\pi^{+} \rightarrow \mu^{+} + \mathrm{v}$ (ii) $\mathrm{p} \rightarrow \mathrm{n} + \mathrm{e}^{+} + \mathrm{v_e}$ (iii) $\mathrm{p} + \pi^{-} \rightarrow \mathrm{K}^{+} + \Sigma^{-}$	TC	15	2013
68.	How are elementary particles classified on the basis of their participation in fundamental interaction?	TC	10	2014
69.	Describe grand unification theories (GUT).	TC	20	2015

70.	How many types of neutrinos exist? How do they differ in their masses?	TC	15	2015 2021
71.	Write down the following decays in terms of quarks: (i) $\Omega^{-} \rightarrow \Lambda^{0} + \mathrm{K}^{-}$ (ii) $\Lambda^{\circ\backslash\mathrm{rightarrowp}} + \pi^{-}$ (iii) $\mathrm{K}^{-} \rightarrow \mu^{\mp}\mathrm{v}_{\mu}$	TC	15	2015
72.	What are elementary particles and how are they classified? Describe in brief the different types of interactions that can occur between the elementary particles.	TC	6 4	2016
73.	Explain the various leptonic family members. What is leptonic number conservation? Based on this conservation law, state whether the following reactions are possible or not: (i) $\pi^{-} \rightarrow \mu^{\mp\bar{v}_{\tau}}$ (ii) $\mathrm{n\backslash rightarrowp}^{+} + \mathrm{e}^{\mp\bar{v}_{\mathrm{e}}}$	TC	10 5 5 5	2016
74.	Write down the quark structure of the following hadrons: $\Delta^{++}, \Omega^{-}, \Sigma^{-\mathrm{and}}\ \Lambda^{\circ}$ Write down the following decays in terms of quarks: (i) $\mathrm{n\backslash rightarrowp}^{+} + \mathrm{e}^{\mp\bar{v}_{\mathrm{e}}}$ (ii) $\Delta^{+} \rightarrow \pi^{+} + \mathrm{n}$ (iii) $\Sigma^{+\backslash\mathrm{rightarrowp}^{+}} + \pi^{0}$	TC	6 9	2016
75.	Explain unification of electromagnetic and weak interactions. What is Z°-boson? What is its relevance in electroweak unification?	TC	10	2016
76.	State the three characteristic properties of strong, weak and electromagnetic forces distinguishing one from the other.	TC	5	2017
77.	Point out the interactions in which the following conservation laws are obeyed or violated (a) Isotopic spin (b) Hyper charge (c) Lepton number (d) Charge conjugation	TC	10	2017
78.	Write down the quark constituents of each of the following: (a) π^{+} (b) K^{+} (c) Δ^{++} (d) Σ° (e) Ω^{-}	TC	5	2017
79.	Which of the following elementary particle reactions/decays are allowed under various conservation laws? If allowed, write down the type of interaction and the characteristic time by which it would proceed : (i) $\mathrm{p} + \mathrm{n} \rightarrow \Lambda^{0} + \Sigma^{+}$ (ii) $\pi^{+} + \mathrm{n} \rightarrow \Lambda^{0} + \mathrm{K}^{+}$	TC	10	2018

	(iii) $p + n \rightarrow K^+ + \Sigma^+$ (iv) $\pi^0 \rightarrow \gamma +$ (v) $\bar{n} \rightarrow \bar{p} + e^+ + v_e$			
80.	Write down the basic weak interaction processes in the nuclei. Also illustrate the beta decays of (i) neutron and (ii) proton.	TC	15	2020
81.	List in two separate columns, the quantities that are conserved and not conserved in the weak interaction of particles.	TC	10	2020
82.	ρ° and K° mesons both decay mostly to $\pi^+$and π^-. Explain why the mean lifetime of ρ° is shorter $(\backsim 10^{-23}\ \mathrm{s})$ compared to the mean lifetime of $K^\circ(\backsim 10^{-10}\ \mathrm{s})$.	TC	10	2021
83.	What are the properties of the particles made up of the following quarks? (a) $u\bar{d}$ (b) $\bar{u}d$ (c) dds (d) uss	TC	10	2021
84.	Explain why each of the following particles cannot exist according to the quark model. (i) A Baryon of spin 1 (ii) An anti-Baryon of electric charge +2	TC	10	2022
85.	Which of the following decays are allowed and which are forbidden? If the decay is allowed, state which interaction is responsible. If it is forbidden, state which conservation law its occurrence would violate. (a) $n \longrightarrow p + e^{\mp\bar{v}_e}$ (b) $\Lambda^\circ \longrightarrow \pi^+ + \pi^-$ (c) $\pi^- \longrightarrow e^\mp$ (d) $\pi^\circ \longrightarrow e^\mp e^+ + v_e + \bar{v}_e$ (e) $\pi^+ \longrightarrow e^\mp e^+ + \mu^+ + v$	TC	15	2022

A/P

SOLID STATE PHYSICS, DEVICES AND ELECTRONICS

An UPSC CSE Physics Optional PYQ Repository

OCTOBER 3, 2023

ABHI PHYSICS

physicsupsc.com

Syllabus

- ✓ Crystalline and amorphous structure of matter; Different crystal systems, space groups;
- ✓ Methods of determination of crystal structure; X-ray diffraction, scanning, and transmission electron microscopies;
- ✓ Band theory of solids - conductors, insulators and semiconductors; Thermal properties of solids, specific heat, Debye theory;
- ✓ Magnetism: para and ferro magnetism; Elements of superconductivity, Meissner effect, Josephson junctions, and applications; Elementary ideas about high-temperature superconductivity.
- ✓ Intrinsic and extrinsic semiconductors; pn-p and n-p-n transistors; Amplifiers and oscillators; Op-amps; FET, JFET, and MOSFET;
- ✓ Digital electronics-Boolean identities, De Morgan's laws, logic gates, and truth tables;
- ✓ Simple logic circuits; Thermostats, solar cells;
- ✓ Fundamentals of microprocessors and digital computers.

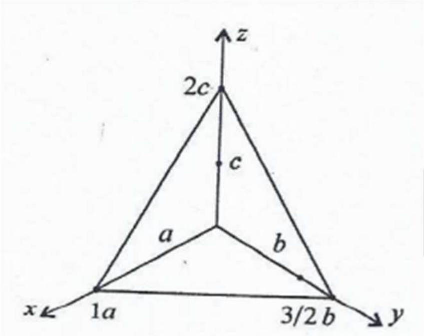
Crystal Structure & its Determination

1. Derive Bragg diffraction condition in vectorial form using incident and diffracted wave vectors and reciprocal lattice vector. TC 10 2011

2. An electron beam of 4 Kev is diffracted through a Bragg angle of 16° for the first maxima. If the energy is increased to 16 keV, find the corresponding Bragg angle for diffraction. AN 10 2011

3. Metallic iron undergoes a structural phase transition from bcc to fcc at $910^\circ\mathrm{C}$. At this temperature, the atomic radii of iron atoms in the bcc and fcc structures are 0.126×10^{-9} m and 0.129×10^{-9} m respectively. Calculate the volume change in percentage during this structural change. AN 12 2012

4. AN 10 2013



A crystal plane is shown in the above figure. Find its Miller indices and interplanar spacing.

5. The velocity of sound in f.c.c. gold and f.c.c. copper is 2100 m/s and 3800 m/s respectively. If the Debye temperature of copper is 348 K, then determine the Debye temperature of gold. Take the densities of gold and copper as $1.93 \times 10^4\ \mathrm{kg/m^3}$ and $0.89 \times 10^4\ \mathrm{kg/m^3}$ respectively. AN 10 2014

6. In a cubic unit cell, find the angle between normals to the plane (111) and (121). AN 10 2014

7. Explain the working of SEM and TEM and highlight the major differences in principles. Draw neat schematic diagrams. TC 20 2014

8. What is the reciprocal lattice and why is it named so? Derive the relationships for the primitive translation vectors of the reciprocal lattice in terms of those of the direct lattice. TC 20 2014

9. Show that any arbitrary rotation axis is not permitted in a crystal lattice. TC 10 2015

10. The primitive translation vectors of a two-dimensional lattice are AN 10 2016

$$a = 2\hat{\imath} + \hat{\jmath}, b = 2\hat{\jmath}.$$

Determine the primitive translation vectors of its reciprocal lattice.

11.	Obtain Laue's equations for X-ray diffraction by crystals. Show that these are consistent with the Bragg's law.	TC	15 5	2016
12.	Calculate Atomic Packing Fraction (APF) for FCC and HCP structures, and show that these are the most closely packed structures.	TC	10	2018
13.	Derive Bragg diffraction law for X-ray diffraction. Compare Laue and Debye-Scherrer methods for crystal structure determination.	TC	10	2018
14.	Deduce the Miller indices of the close-packed planes of atoms in the f.c.c. lattice.	TC	10	2019
15.	The angles between the tetrahedral bonds of diamond are the same as the angles between the body diagonals of a stack of neighbouring cubes having common edges and not faces. Use vector analysis to find the value of the angle.	TC	10	2019
16.	Consider a face-centred cubic lattice of side a. Deduce – (i) The primitive translation vectors; (ii) The volume of the primitive cell; (iii) The reciprocal primitive translation vectors; (iv) The volume of the reciprocal lattice	TC	20	2019
17.	The lattice parameter and the atomic mass of a diamond crystal are 3.57Å and 12 , respectively. Calculate the density of the crystal. Given, Avogadro's number, $N = 6.023 \times 10^{26} (\mathrm{~kg~mol})^{-1}$.	AN	10	2020
18.	In an almost pure thick aluminium sheet, there are 0.19 atomic percent of copper at the surface and 0.18 atomic percent at a depth of $1.2\ \mathrm{mm}$ from the surface. Calculate the flux of the copper atoms from the surface at $550^{\circ}\mathrm{C}$, if the diffusion coefficient of copper in aluminium at this temperature is $5.25 \times 10^{-13}\ \mathrm{m}^2\ \mathrm{s}^{-1}$. Given, Al FCC with lattice parameter, $a = 4.05\text{Å}$.	AN	10	2020
19.	Calculate the Hall coefficient of sodium based on the free electron model. Sodium has BCC structure and the side of the cube is 4.28Å.	AN	10	2020
20.	An X-ray beam of wavelength (λ_1) undergoes a first order Bragg reflection at a Bragg angle of 30°. X-ray of wavelength $97\ \mathrm{nm}$ undergoes $3\mathrm{rd}$ order reflection at a Bragg angle of 60°. Consider that the two beams are reflected from the same set of planes. Find the value of λ_1.	AN	10	2021
21.	Considering atoms hard, uniform spheres, find the number of atoms per unit cell and packing fraction for simple cubic, bcc and fcc structures.	TC	15	2022
22.	The wavelength of a prominent X-ray line from a copper target is $0.1512\ \mathrm{m}$. The radiation, when diffracted with (111) plane of a crystal with fcc structure, corresponded to a Bragg angle of 20.2°. If the density of the crystal is $2698\ \mathrm{kg/m^3}$ and atomic weight is $26.98\ \mathrm{kg/kmol}$, calculate the Avogadro number.	AN	15	2022

Band *theory of solids*

23.	Explain the reduced zone scheme of plotting the energy bands in solids. On the basis of E vs k curve distinguish between conductors, semiconductors and insulators.	TC	10	2010
24.	What is nearly free electron approximation? On the basis of this approximation explain the formation of energy bands in solids.	TC	20	2010
25.	Explain the origin of energy band formation in solids. Show that in nearly free electron approximation, the energy band gap is $2\|V_G\|$, where V_G is Fourier transform of periodic potential seen by the valence electrons.	TC	20	2011
26.	Starting with the expression for the density of states for electrons in a band, show that the Fermi energy of an intrinsic semiconductor is at the middle of the band gap. Use these results to estimate the electron density at $300 \mathrm{~K}$ (Assuming $\mathrm{E_g} = 1\mathrm{eV}$ and the rest masses of electron and hole as $\mathrm{m_e}$ and $\mathrm{m_h}$).	AN	20	2013
27.	How does the energy gap in superconductors differ from the energy gap in insulator? How does it vary with temperature for superconductors?	TC	10	2014
28.	Describe the motion of an electron in one dimensional periodic potential and show that it leads to formation of bands of allowed and forbidden states in the electron energy spectrum. How are the conductors, semiconductors and insulators discriminated on the basis of band structure?	TC	20	2015
29.	The energy (E) and wave vector (k) for a conduction band electron in a semiconductor are related as $E = \alpha \frac{\hbar^2 k^2}{m_0}$ where α is a constant and m_0 is the free electron mass. Calculate the effective mass of the electron.	TC	10	2017
30.	Derive the expression for the average energy of a quantum oscillation of frequency v. Assume Fermi-Dirac distribution and $E - E_{\mathrm{F}} > 2$, where E_{F} is the Fermi level.	TC	15	2019
31.	Show that for an n-type semiconductor, the Fermi level lies midway between the donor states and the conduction band edge at low temperature (assuming $E_v = 0$).	TC	20	2022

Thermal and Magnetic Properties of Solids

32.	Distinguish between Einstein and Debye models of specific heats of solids. Obtain an expression for the specific heat of a solid on the basis of Debye model. Discuss the results for low and high temperature ranges.	TC	20	2010
33.	Distinguish between classical and quantum theory of paramagnetism. Explain the paramagnetism of free electrons on the basis of F.D. distribution.	TC	20	2010
34.	Find an expression for lattice specific heat of solids, and its low and high temperature limits. What is Debye temperature?	TC	20	2011
35.	Classify diamagnetic, paramagnetic and ferromagnetic materials in terms of their magnetic susceptibility (ψ). Plot and explain the variation of $\frac{1}{\psi}$ with temperature for the three materials.	TC	15	2012
36.	Find an expression for lattice specific heat of a solid, and its low and high temperature limits. What is Debye temperature?	TC	20	2015
37.	With the help of a schematic diagram, show how entropy and specific heat vary with temperature for a superconductor.	TC	10	2016
38.	Write down the salient features of the Einstein's theory of lattice heat capacity. Further write down the expression for specific heat in Einstein's theory and explain its high and low temperature limits.	TC	4 6	2016
39.	A system of paramagnetic atoms (N per unit volume) which can occupy only two energy levels in a uniform external magnetic field H is at a temperature T. If this system follows Boltzman's distribution, find the magnetisation and susceptibility of the system.	TC	20	2017
40.	Derive an expression for lattice specific heat in Debye model. Find its low temperature limit (Debye T^3 law).	TC	15	2018
41.	Using the expression for internal energy $U = 3N\frac{\hbar\omega}{e^{\hbar\omega/k_BT}-1}$, show that Einstein specific heat capacity is given by; $$C = 3R\left(\frac{\hbar\omega}{k_BT}\right)^2 \frac{e^{\hbar\omega/k_BT}}{(e^{\hbar\omega/k_BT}-1)^2}$$ Also show that Einstein specific heat capacity given above is proportional to $e^{-\hbar\omega/k_BT}$ at very low temperature.	TC	10	2021
42.	A solid contains a dilute concentration of Nd^{3+} ions, each of which possess three $4f$ electrons. Assuming that there are $10^{25}\ \mathrm{m}^{-3}$ of these ions, calculate the magnetic susceptibility of the sample at $1K$.	AN	20	2021
43.	Explain the drawbacks of Einstein's theory of specific heat and how it was overcome by Debye.	TC	20	2022

Superconductivity

44.	What is Josephson tunnelling? Distinguish between a.c. and d.c. Josephson effects	TC	10	2010

45.	Describe the characteristic properties of a superconductor. Derive London equation for a superconductor and hence explain Meissner effect.	TC	20	2011
46.	Explain the variation of Magnetisation of Type I and Type II superconductors as a function of applied magnetic field	TC	15	2012
47.	A perfect conductor and a superconductor with $T_c \sim 10\ \mathrm{K}$ are subjected to the following conditions: (i) cooled under applied magnetic field to $4\ \mathrm{K}$ (ii) cooled to $4K$ and then magnetic field is applied with schematic diagrams, explain path of magnetic field lines in all these situations.	TC	15	2013
48.	Show that the London equation $\vec{\nabla} \times \vec{J} = -\frac{1}{\mu_0 \lambda_L^2} \vec{B}$ or $\vec{j} = -\frac{C}{4\pi\lambda_L^2} \vec{A}$ leads to the Meissner effect.	TC	15	2013
49.	Lead in the superconducting state has critical temperature of $6.2\ \mathrm{K}$ at zero magnetic field and a critical field of $0.064\mathrm{MAm}^{-1}$ at $0\ \mathrm{K}$. Determine the critical field at $4\ \mathrm{K}$.	AN	10	2014
50.	Distinguish between a superconductor and perfect conductor. Explain what is a Cooper pair.	TC	10	2015
51.	Obtain the expression for penetration depth using London's equation of superconductivity and explain its significance.	TC	10	2017

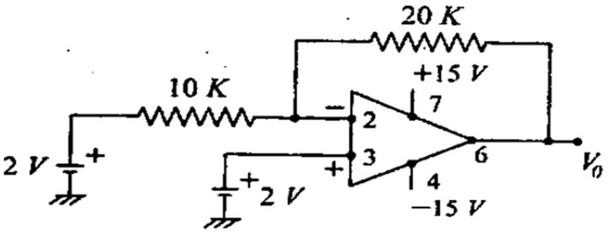
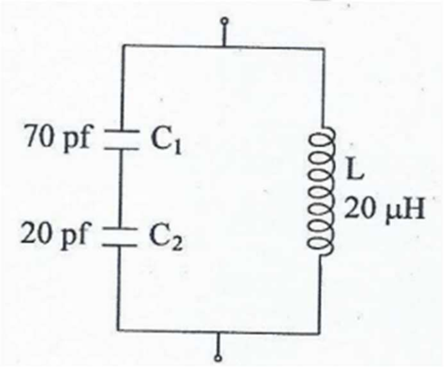
52.	What are type I and type II superconductors? Give examples. Discuss and compare Meissner effect and perfect diamagnetic behaviour for type I and type II superconductors.	TC	20	2018
53.	Using the two-fluid model of a conductor (normal and superconducting) and the Maxwell's equations, derive the two London equations of superconductivity.	TC	20	2020
54.	Explain why Type-II superconductor is better than Type-I superconductor in the application of superconductor magnets.	TC	10	2022
55.	Compare the dependence of resistance on temperature of a superconductor with that of a normal conductor. Describe briefly the formation of Cooper pairs.	TC	15	2022

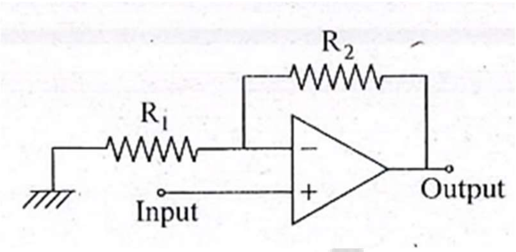
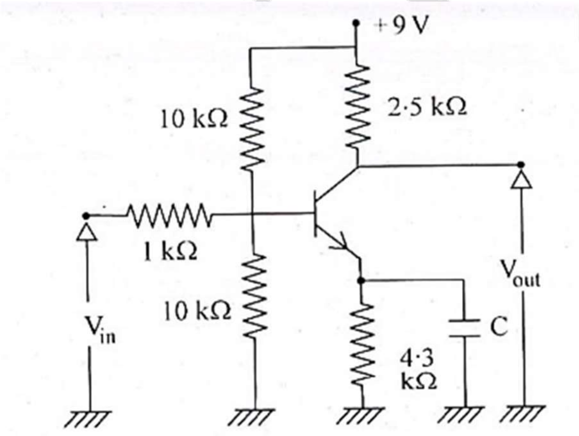
Semiconductors/Solar cell

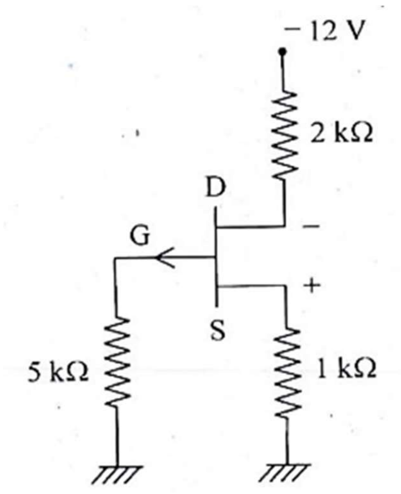
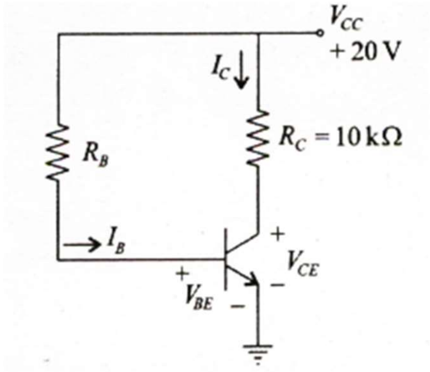
56.	Distinguish between intrinsic and extrinsic semiconductors. Show that in a semiconductor, the product of concentrations of the two types of charge carriers is constant at a given temperature.	TC	30	2011
57.	Starting from an expression for the group velocity, arrive at an expression for the effective mass of an electron in a semiconductor. Explain the significance of sign of the effective mass.	TC	30	2012
58.	An electric field of $100\ \mathrm{V/m}$ is applied to a sample of n-type semiconductor whose Hall coefficient is $-0.0125\ \mathrm{m}^3$/ coulomb. Determine the current density in the sample assuming $\mu_{\mathrm{x}} = 0.36\ \mathrm{m}^2\mathrm{v}^{-1}\ \mathrm{s}^{-1}$.	AN	10	2014

59.	Draw the device structure of a p-n junction solar cell and explain how light energy is converted into electrical energy. Draw and explain its I-V characteristics.	TC	15	2015
60.	What is the difference between direct and indirect band gap semiconductors? Which one is suitable for use in solar cells?	TC	10	2016
			10	
61.	In a semiconductor, the effective masses of an electron and a hole are $0.07\ \mathrm{m}_0$ and $0.4\ \mathrm{m}_0$, respectively, where m_0 is the free electron mass. Assuming that the average relaxation time for the hole is half of that for the electrons, calculate the mobility of the holes when the mobility of the electrons is $0.8\ \mathrm{m}^2$ volt $^{-1}\ \mathrm{s}^{-1}$.	AN	15	2017
62.	What are intrinsic and extrinsic semiconductors? Show that in the intrinsic semiconductors, Fermi level lies exactly in the middle of bottom of conduction band and top of valence band.	TC	15	2018
63.	A silicon semiconductor sample at $T = 300\ \mathrm{K}$ having cross-sectional area of $0.5\mu\mathrm{m}^2$ has a pentavalent donor doping profile given by $C(x) = 5 \times 10^{16} e^{(-x/L_n)}\mathrm{cm}^{-3}$. Given, the mobility of the electrons in the sample is $1250\ \mathrm{cm}^2\ \mathrm{V}^{-1}\ \mathrm{s}^{-1}$ and the diffusion length of the electrons, L_n, is $4\mu\mathrm{m}$. Calculate the diffusion current in the sample at distance $x = 2\mu\mathrm{m}$.	AN	10	2019
64.	A silicon semiconductor sample is doped with $6 \times 10^{16}\ \mathrm{cm}^{-3}$ of aluminium and $7 \times 10^{15}\ \mathrm{cm}^{-3}$ of phosphorus atoms. Given at $T = 300\ \mathrm{K}$, the intrinsic carrier concentration, $n_i = 1.5 \times 10^{10}\ \mathrm{cm}^{-3}$; the band gap, $E_g = 1.1\mathrm{eV}$; the electron mobility, $\mu_n = 1250\ \mathrm{cm}^2\ \mathrm{V}^{-1}\ \mathrm{s}^{-1}$ and the hole mobility, $\mu_p = 480\ \mathrm{cm}^2\ \mathrm{V}^{-1}\ \mathrm{s}^{-1}$. Determine in the sample of the following: (i) The type of the semiconductor, n or p (ii) The hole carrier concentration (iii) The electron carrier concentration (iv) The position of the Fermi level in the sample with respect to the bottom of the conduction band (v) The conductivity of the sample	AN	20	2019
65.	A $5\ \mathrm{cm}^2\mathrm{Ge}$ solar cell with a dark reverse saturation current of $2\mathrm{nA}$ has solar radiation incident upon it, producing 4×10^{17} electron-hole pairs per second. The electron and hole diffusion lengths are given to be $5\mu\mathrm{m}$ and $2\mu\mathrm{m}$, respectively. Calculate for the cell of the following: (i) The short-circuit current (ii) The open-circuit voltage	AN	10	2019
66.	What is an intrinsic semiconductor? Intrinsic silicon has a band gap of $1.1\mathrm{eV}$ and yet at $T = 300\ \mathrm{K}$, the conductivity is non-zero. Explain. Comment, with the help of relevant expression, on the position of the Fermi level of an intrinsic semiconductor.	AN	15	2020

TRANSISTORS (Amplifiers/Oscillators)

67.	 Identify the above op-amp circuit. Estimate the net output voltage V_0	AN	10	2010
68.	Draw a two stage RC coupled transistor amplifier. Plot the frequency response and indicate the lower and upper cut off frequencies and the bandwidth.	TC	30	2010
69.	Give the basic structure of a n-channel depletion type MOSFET. Draw the drain current-drain voltage characteristics both in depletion as well as in enhancement modes.	TC	20	2010, 2014
70.	Draw the common-base amplifier circuit, using an n-p-n transistor and briefly discuss its working.	TC	15	2011
71.	What are the rules followed in biasing a bipolar junction transistor when it is used as an amplifier? Show how n-p-n and p-n-p transistors to be used as an amplifier are biased and the direction of flow of base, collector and emitter currents.	TC	12	2012
72.	Construct an operational amplifier based analog computer to solve the differential equation $$\frac{d^2v}{dt^2}-\frac{dv}{dt}+2v=0$$	TC	30	2012
73.	Draw and explain the collector characteristics of a bipolar junction transistor in common emitter configuration. Using the plot, explain how the transistor can be used as an ON-OFF switch.	TC	10	2013
74.	 Explain how the circuit shown above can be a source of oscillations. Use this circuit to construct a transistor oscillator and explain its working. What is the frequency of oscillations of this circuit?	TC	25	2013
75.	Differentiate between n-p-n and p-n-p transistors. Give their device structure and biasing circuits when used as an amplifier.	TC	10	2015

76.	Design a transistor based Colpitt oscillator which can oscillate at 9MHz. Explain how the oscillations are created and sustained.	AN	15	2015
77.	Describe an operational amplifier based integrator. Using operational amplifier integrators, design a circuit to solve the following differential equation: $\frac{d^2v}{dt^2}+2\frac{dv}{dt}+3v=0$	TC	20	2015
78.	 Consider the operational amplifier circuit given above: Given, $R_1 = 10\text{k}\Omega, R_2 = 150\text{k}\Omega$ and the product of the open loop gain of the amplifier and its bandwidth $= 10^6$ Hz. Determine the closed loop bandwidth of the amplifier.	AN	10	2017
79.	 An amplifier in common-emitter configuration is shown in the above figure. If the current gain $\beta = 100$ and the a.c. emitter resistance $= 25.0\Omega$, determine the input impedance and the voltage gain of the amplifier:	AN	15	2017

80.	 Given above is a circuit of self biased p-channel JFET. If the pinch off voltage is $5.0\ \mathrm{V}$ and $\mathrm{V_{DS}} = 6.0\ \mathrm{V}$, calculate the saturation current $\mathrm{I_{DSS}}$.	AN	15	2017
81.	What are operational amplifiers? How can it be used as an inductor? Prove it mathematically.	TC	15	2018
82.	Sketch the cross-sectional structure of an enhancement mode MOSFET and explain its principles of operation with the help of its output characteristics.	TC	15	2019
83.	Consider an amplifier with an open-loop (no feedback) gain of A and a feedback factor β. Derive the expression for the gain with feedback, A_f. Derive the condition for the amplifier with feedback to act as an oscillator. Comment on the change in A_f with a change in A.	TC	15	2020
84.	An $n-p-n$ transistor with $\beta = 49$ is used in common-emitter amplifier mode with $V_{cc} = 10\ \mathrm{V}$ and $R_L = 2\mathrm{k\Omega}$. If a $100\mathrm{k\Omega}$ resistor is connected between the collector and the base of the transistor, calculate the quiescent collector current. Assume $V_{BE} = 0$.	AN	20	2021
85.	Calculate the pinch-off voltage for n-channel silicon FET with a channel width of $6 \times 10^{-4}\ \mathrm{cm}$ and a donor concentration of $10^{15}\ \mathrm{cm^{-3}}$. Given that dielectric constant of silicon is 12 .	AN	10	2021
86.		AN	10	2021

Sketch the dc load line for the circuit shown.

87.	Why is the Field Effect Transistor (FET) called Unipolar Transistor? Discuss how it is superior than Bipolar Junction Transistor.	TC	10	2022

Digital Electronics/Microprocessors

88.	Why are NAND and NOR gates called universal gates? Give the logic diagram, Boolean equation and the truth table of NAND gate. Design an OR gate using only NAND gates.	TC	15	2010,2016
89.	Simplify the logical expression $AB + \bar{A}\bar{B} + ABC$ using a Karnaugh map.	TC	10	2011
90.	Simplify the logical expression $\overline{(A + \bar{B})}B(A + \bar{C})$ and draw the logical circuit to implement it.	TC	15	2011
91.	Explain the race problem encountered in an RS latch and how it can be avoided. Build a Mod-10 counter using JK flip-flops and explain its working.	TC	30	2012
92.	Construct a digital circuit to add three bits A, B and C and provide their sum and carry as outputs. Show appropriate Boolean expressions and truth table to justify the outputs.	TC	25	2013
93.	Simplify the logical expression $[A\bar{B}(C + BD) + \bar{A}\bar{B}]C$.	TC	10	2015
94.	Describe the working of a microprocessor system in block diagram. How is its performance affected in a pipelined processor?	TC	15	2018
95.	Why NAND and NOR gates are called universal gates? Give the logic diagram, Boolean equation and the truth table of a $\mathrm{X} - \mathrm{OR}$ gate.	TC	10	2022

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UPSC CSE Main 2023 paper 1

Section A

1. a) A force $\vec{F}$ is given by $\vec{F} = x^2y\hat{x} + zy^2\hat{y} + xz^2\hat{z}$. Determine whether or not the force is conservative. 10

b) Calculate the gravitational self-energy of the Earth. 10
Given:

Mass of Earth $M_e = 6 \times 10^{24}\ kg$ and the Radius of Earth $R_e = 6.4 \times 10^6\ m$ 10

c) What are the consequences of Lorentz transformations on length and time when observed from a frame moving at relativistic velocities? 10

d) Using Huygens' principle for a plane wave travelling from rarer medium 1 to a denser medium 2, show that 10

$$\frac{sin\, i}{sin\, r} = \frac{v_1}{v_2} = \frac{\mu_2}{\mu_1},$$

where i and r are the angles of incidence and refraction, respectively. v_1, μ_1 and v_2, μ_2 are the velocities and refractive indices in media 1 and 2, respectively.

e) What are three and four level pumping schemes? Explain the lasing action in these with schematic diagrams. 10

2. a) (i) Derive the expressions for gravitational potentials at a point 10
(I) outside the spherical shell
(II) inside the spherical shell

(ii) Calculate the escape velocity of a body of mass $10\ kg$ from the surface of Moon $\left(g_{\text{Moon}} = \frac{1}{6}\, g_{\text{Earth}}\right)$.
Mass of Moon $= 7.3 \times 10^{22}\ kg$ 10
Radius of Moon $= 1.7 \times 10^6\ m$

b) Obtain condition for achromatism of two thin lenses separated by a finite distance. If the dispersive powers of the materials of the two lenses are 0.020 and 0.028, their focal lengths are $10\ cm$ and $5\ cm$, respectively. Calculate the separation between them in order to form achromatic combination. 15

c) (i) The quantities of rotatory motion are analogous to those of translatory motion. Write the corresponding equations of translatory and rotatory motion. 5

(ii) Describe the theorems of perpendicular and parallel axes in case of a plane lamina 10

3. a) (i) What are the requisite conditions for observation of interference pattern on a screen? 5

(ii) Derive the expression for fringe width and intensity at a point on the screen in a double slit experiment. 10

b) (i) Prove that the separation of two colliding particles is same, when observed in centre of mass and laboratory system. 10

(ii) Determine the kinetic energy of a thin disc of mass 0.5 kg and radius 0.2 m rotating with 100 rotations per second around the axis passing through its centre and perpendicular to its plane. 5

c) Write equation for damped harmonic oscillations and obtain expression for logarithmic decrement.

In a damped harmonic motion, the first amplitude is 10 cm, which reduces to 2 cm after 50 oscillations, each of period 4 seconds. Determine the logarithmic decrement. Also, calculate the number of oscillations in which the amplitude decreases to 25%. 20

4. a) Write conditions for working of a step-index optical fiber. In a step-index fiber, the core and cladding materials have refractive indices 1.50 and 1.43, respectively. 20
Find the following:
i. Critical propagation angle
ii. Acceptance angle
iii. Total time delay in 1 km length of the fiber
iv. Total dispersion in 50 km length of the fiber

b) Define streamline flow of a fluid. Using the equation of continuity for an isotropic fluid, find different components of total energy per unit volume. 15

c) (i) What is the difference between Fresnel diffraction and Fraunhofer diffraction? 5

(ii)What is resolving power of a telescope? Why is the resolving power of microscope more with UV light than with visible light. 10

Section B

5. a) Find the energy stored in a system of four charges $Q_1 = 1nC, \mathrm{Q}_2 = 2nC$, $Q_3 = 3nC$ and $Q_4 = 4nC$ placed at the cartesian coordinates $R_1(1,1)$, $R_2(2,1)$, $R_3(1,4)$ and $R_4(2,2)$, respectively. Assume free space. 10

b) Derive the expression for the inductance per unit length of two long parallel wires each of radius a, separated by distance d from their axes and carrying equal and opposite current I. 10

c) Show that Continuity equation is embedded in Maxwell's equations. 10

d) Using Zeroth law of thermodynamics, introduce the concept of temperature. Explain how the isotherms of two different systems can be drawn. 10

e) Write down the expressions for the Fermi-Dirac distribution and the Bose-Einstein distribution. Plot the distributions as a function of the energy. 10

6. a) Two inductors having inductances L_1 and L_2 are connected in parallel. The inductors have a mutual inductance M. Derive the expression for the effective inductance. Assume the inductors have negligible resistances. 15

b) (i) Define Joule-Kelvin coefficient. Write it in its mathematical form. 5

(ii) Determine the Joule-Kelvin coefficient for a van der Waals gas. Hence, obtain an expression for temperature of inversion. Discuss the conditions under which heating or cooling is produced. 10

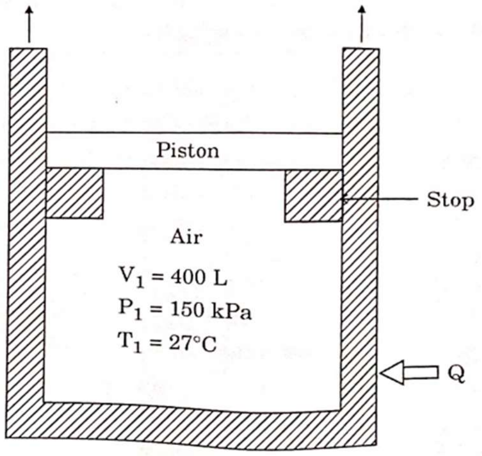
c) Consider the interaction of an electromagnetic wave at the interface of two dielectric media. If electric field $\vec{E}$ is parallel to the plane of incidence, obtain Fresnel's equations and Brewster's law of polarization. 20

7. a) A neutral atom consists of a point nucleus $+q$ surrounded by a uniformly charged spherical cloud $(-q)$ of radius r. Show that when such an atom is placed in a weak external electric field $\vec{E}$, the atomic polarizability of the atom is proportional to the volume of the sphere. 15

b) A piston-cylinder device initially contains air at $150kPa$ and $27°C$. At this state, the piston is resting on a pair of stops, as shown in the figure, and the enclosed volume is 400 L. The mass of the piston is such that a $350kPa$ pressure is required to move it. The air is now heated until the volume is doubled. Determine: 20

a. the final temperature,
b. the work done by the air, and
c. the total heat transferred to air

Given:
$U_{300\text{ K}} = 214\ kJ/kg$ and $U_{\text{final}} = 1113\ kJ/kg$
i. Gas constant of air, $R = 0.287kPa.\text{m}^3/kg.K$



c) A spherical shell of radius R, carrying a uniform surface charge σ, is set spinning at angular velocity ω about its axis. Find the vector potential it produces at point $\vec{r}$. 15

8. a) A circular ring of radius R lying on the x$-y$ plane and centred at the origin, carries a uniform line charge λ. Find the first three terms (monopole, dipole and quadrupole) of the multipole expansion of potential $V(r, \theta)$. 20

b) Two charges $Q_1 = 3nC$ and $Q_2 = 4nC$ are placed at the cartesian points $(0,2,2)m$ and $(0, -2,4)m$, respectively. The $z = 0$ plane is connected to the ground. Calculate the electric potential and the electric field at the point $(3,2,4)m$ using the method of images. 15

c) Use the Maxwell-Boltzmann distribution to find the number of oxygen molecules whose velocities lie between 195 m/s and 205 m/s at $0^{\circ}C$. The given mass of oxygen gas is 0.1 kg. (Assume mass of proton to be 1.66×10^{-27} kg) 15

UPSC CSE Main 2023 – Paper 2 – Physics Optional

Section A

1 (a)	Calculate the zero-point energy for a particle in an infinite potential well for the following cases: (i) a 100 g ball confined on a 5 m long line. (ii) an oxygen atom confined to a 2×10^{-1} m lattice. (iii) an electron confined to a 10^{-10} m atom. Why zero point energy is not important for macroscopic objects? Comment.	10
1 (b)	Consider a particle of mass m and charge q moving under the influence of a one dimensional harmonic oscillator potential. Assume it is placed in a constant electric field E. The Hamiltonian of this particle is therefore given by $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 X^2 - qEX$. Obtain the energy expression and the wave function of the nth excited state of the particle.	10
1 (c)	A particle of mass m is in a spherically symmetric attractive potential of radius a. Find the minimum depth of the potential needed to have two bound states of zero angular momentum.	10
1 (d)	A beam of hydrogen atoms emitted from an oven at 400k is sent through a Stern-Gerlach experiment having magnet of length 1 m and a gradient field of 10tesla/m. Calculate the transverse deflection of an atom at the point where the beam leaves the magnet.	10
1 (e)	If an atom is placed in a magnetic field of strength 0.1 weber $/\mathrm{m}^2$, then calculate the rate of precession and torque on an electron with $l = 3$ in the atom. Given that the magnetic moment of the electron makes an angle of 30° with the magnetic field.	10
2 (a)	An operator P describing the interaction of two spin $\frac{1}{2}$ particles is $P = a + b\vec{\sigma}_1 \cdot \vec{\sigma}_2$, where a and b are constants, and $\vec{\sigma}_1$ and $\vec{\sigma}_2$ are Pauli matrices of the two spins. The total spin angular momentum $\vec{S} = \vec{S}_1 + \vec{S}_2 = \frac{1}{2}\hbar(\vec{\sigma}_1 + \vec{\sigma}_2)$. Show that P, S^2 and S_Z can be measured simultaneously.	15
2 (b)	Consider a stream of particles of mass m each moving in the positive x-direction with kinetic energy E towards the potential barrier	15

$$V(x) = 0 \quad \text{for} \quad x \leqslant 0$$
$$V(x) = \frac{3E}{4} \quad \text{for} \quad x > 0$$

Find the fraction of particles reflected at $x = 0$.

2 (c)	Consider the potential $V(x) = \begin{cases} 0, & 0 < x < a \\ \infty, & \text{elsewhere} \end{cases}$ (a) Estimate the energies of the ground state as well as those of the first and the second excited states for (i) an electron enclosed in a box of size $a = 10^{-10}$ m. (ii) a 1 g metallic sphere which is moving in a box of size $a =$ 10 cm. (b) Discuss the importance of the Quantum effects for both of these systems. (c) Estimate the velocities of the electron and the metallic sphere using uncertainty principle.	20
3 (a)	What is vector atom model? How the principal features of vector atom model were explained by Stern-Gerlach experiment?	5+10
3 (b)	What is Lande's g factor? Evaluate the Lande's g factor for the $^3P_1$ level in the $2p3s$ configuration of the ^{6}C atom. Also calculate the splitting of the level when the atom is placed in an external magnetic field of 0.1 tesla.	5+5+5
3 (c)	What is Raman effect? Explain Quantum theory of Raman effect and Rotational Structure of a Raman spectrum.	5+10+5
4 (a)	A particle constrained to move along x-axis in the domain $0 \leqslant x \leqslant L$ has a wave function $\psi(x) = \sin\left(\frac{n\pi x}{L}\right)$, where n is an integer. Normalize the wave function and evaluate the expectation value of momentum of the particle.	15
4 (b)	Evaluate the most probable distance of the electron of the hydrogen atom in its $2p$ state. What is the radial probability density at that distance?	15
4 (c)	What is nuclear magnetic resonance? Explain its working principle and use in magnetic resonance imaging systems.	5+5+10

Section B

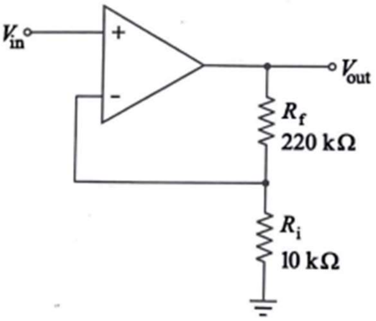
5 (a)	How could you establish that v_e and $\bar{v}_e$ are two different particles?	10
5 (b)	What is the age of a fossil that contains 6 g of carbon ^{14}C and has a decay rate of 27 decays per minute? Given : Ratio $\frac{^{14}C}{^{12}C} = 1 \cdot 3 \times 10^{-13}$, Half life $\left(T_{1/2}\right)$ of $^{14}C = 5730$ yrs.	10
5 (c)	(i) $K^{+} \longrightarrow \Pi^{+} + \Pi^{+} + \Pi^{-}$ (ii) $\Pi^{+} + \rho \longrightarrow \Pi^{+} + \Pi^{+} + n$ (iii) $\Pi^{+} + \rho \longrightarrow \Delta^{++} \longrightarrow \Pi^{+} + \rho$ (iv) $\Sigma^{\circ} \longrightarrow \lambda^{\circ} + \gamma$ (v) $\Sigma^{+} \longrightarrow \lambda^{\circ} + e^{+} + v_e$ (vi) $K^{-} + \rho \longrightarrow K^{+} + K^{\circ} + \Omega^{-}$ (vii) $\Pi^{\circ} \longrightarrow \gamma + e^{+} + e^{-}$ (viii) $\Sigma^{-} \longrightarrow n + e^{-} + \bar{v}_e$ (ix) $\lambda^{\circ} \longrightarrow \rho + e^{-} + \bar{v}_e$ (x) $e^{+} + e^{-} \longrightarrow \gamma + \gamma$ Name the interactions via which the above nuclear decays occur:	10
5 (d)	Derive diffraction conditions using reciprocal lattice concept. What are these conditions known as?	10
5 (e)	Show that the Fermi level shifts upward, closer to the conduction band in an n-type semiconductor and shifts downward, closer to the valence band in a p-type semiconductor.	10
6 (a)	Establish the Rutherford's scattering cross section formula for α-particle by considering the standard assumptions and symbols.	20
6 (b)	By assuming the nucleus as a cubical box of length equal to the nuclear diameter 10^{-12} cm, calculate the kinetic energy of the highest level occupied nucleon of iron-56 nucleus.	15
6 (c)	What do you understand by nuclear forces? Explain meson theory of exchange forces.	5+10
7 (a)	Explain classical theory of diamagnetism. Show that the susceptibility of diamagnetic substances is directly proportional to the atomic number. Why all the electrons in an atom contribute to diamagnetism?	5+8+2
7 (b)	Derive an expression for the specific heat of a solid based on the Debye theory and show how it agrees with the experimental values. What is the most important assumption of Debye theory in comparison to Einstein theory? Is there any drawback of Debye theory?	15+3+2

7 (c) With a neat circuit diagram, explain the working of Wien-Bridge oscillator. 15

8 (a) What do you understand by the critical size of a reactor? Explain the main features of nuclear reactors. 5+15

8 (b) What is superconductivity? Explain Meissner effect. Why superconductors should be a diamagnetic material? 15

8 (c) 10+5



(i) Determine the input and output impedances of the amplifier in given figure. The op-amp datasheet gives $Z_{in} = 2\text{M}\Omega, Z_{out} = 75\Omega$ and $A_{ol} = 200{,}000$ (open loop voltage gain).
(ii) Find the closed-loop voltage gain.