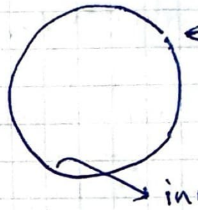
1. THE ORIGIN OF QUANTUM MECHANICS

① Blackbody Radiation : Planck's Quantum Hypothesis

B.B → its study led to begening of conceptual fabrication of Q.M.

→ nearest approximation (as ideal B.B doesn't exist) : a hollow enclosure which has a small entrance hole, and, blackend inner walls.

↓ any object which absorb all of the thrown rediation on it.



1.1 Spectrum emitted by B.B

Fig 1 → depicts the original spectrum as emitted by Blackbody.

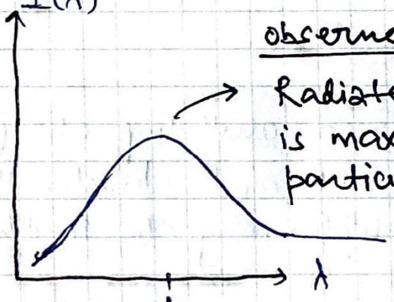


Fig. 1 → Spectral distribution of Radiation from Blackbody.

Hence, many theorist tried to explain the emitted spectrum (Fig. 1).

Some prominent theories were → Wein's law
→ Rayleigh Jeans law
→ Planck's Radiation law.

1.2 Wein's Law

- used thermodynamics + dimensional ~~and~~ Analysis and scaling arguments.
- He proposed → $I(\lambda, T) = \frac{a e^{-b/\lambda T}}{\lambda^5}$
  ↳ intensity as a function of wavelength & temperature
- Such law fit the observed curve well, but failed at longer wavelengths.

1.3 Rayleigh-Jeans law

- Wein's formula was semi-empirical and was not derived from known and well established theory.
- Hence, next theory was given by Rayleigh and Jeans, which were based on classical electrodynamics & thermodynamics.
- They assumed Radiating body as :- Collection of large number of charged particles.

↓

What they are doing?

↓

They are performing linear S.H.O.

← These oscillating charges do emit and absorb the electro-magnetic radiation.

- At thermal equilibrium

↓

energy density of radiation within the cavity = energy density of the atomic oscillators at the walls of the cavity.

Mathematically, it was discovered that no. of such oscillators per unit volume is $n(\nu) = \frac{8\pi c^2}{\nu^3}$

where ν is frequency.

And for such setup the average energy given by classical equipartition theorem is KT.

- Hence energy density of the radiation of the frequency ν is → $U(\nu, T) = n(\nu) \cdot \langle E \rangle$

$$\boxed{U(\nu, T) = \frac{8\pi c^2}{\nu^3} \cdot KT}$$

→ But failed at shorter wavelength while matching observed results

Rayleigh-Jean's Law.

and mathemeticelly, one can write the Rayleigh-Jeans law ~~the~~ in the term of wavelength :-

$$n(\nu)\,d\nu = n(\lambda)\,d\lambda$$

$$n(\lambda)\,d\lambda = n(\nu).\,d\nu$$

$$\Rightarrow \quad n(\lambda) = n(\nu)\cdot\frac{d\nu}{d\lambda}$$

$$= \frac{8\pi\nu^2}{c^3}\cdot\frac{d}{d\lambda}\left(\frac{c}{\lambda}\right)$$

$$= \frac{8\pi}{\lambda^4}$$

Hence, $U(\lambda,T) = \frac{8\pi}{\lambda^4}.\,KT$

Furthermore, the intensity of radiation of cavity hole is proportional to $U(\lambda,T)$. Although the exact relation is -

$$I(\lambda) = \frac{c}{4}.\,U(\lambda,T).$$

→ Such arguments are necessary in order to improve ANSWER Quality.

But its main failure, leads to total energy in the freqnency range 0 to ∞ to infinite.

$$U(T) = \int_0^\infty U(\nu,T)\,d\nu = \frac{8\pi\nu^2}{c^3}.\,KT.\,d\nu = \infty$$

But in any range of freqnency there can not be infinite energy. This issue was well known as UV catastrope.

→ Asked as PYQ.

Fundamental Flaw of Rayleigh Jeans law :-

- Energy goes to infinite a.k.a UV catastrophe
- As this law was derived using concepts from classicel Mechanics. So its failure leads to conclusion that study / development of (C.P) is incomplete.

(C.P → classicel physics.)

1.4 Planck's Radiation Law

To solve the underlying issue of explanation of the spectrum observed ~~by~~ of a Blackbody, Planck assumed that the —

Quantisation of Energy ⇑

$$E = nh\nu$$

where $n = 0, 1, 2, 3..$

Material oscillator (with walls of cavity) can only have discrete energy, rather than the continous range of energies. If a particle is oscillating with ~~energy~~ frequency ν, then energy can have values which are integral multiples of $h\nu$ (h → Planck's constant, ν → frequency)

↓

As energies are integral multiple of $h\nu$, hence their average energy can not be KT. It has to be something else.

Calculation of Average Energy considering Quantisation of energy :-

- consider all particles are oscillating with frequency ν.
- At, absolute zero → all the oscillators will be in ground state.
- At, rising temperature → Some of the oscillators will jump to excited state / higher energy levels.
- Thus at temp. T, in equilibrium, the no. of oscillators with energy E_n will be :-

$$N = N_0 \cdot e^{-E_n/KT}$$ } as per Maxwell-Boltzman statistics. ①

As per the eq①, we can see that as $n \to \infty$

$N \to 0$ and hence higher energy states are less likely to be populated.

And average energy per oscillator is –

$$\overline{E_n} = \frac{\sum_{n=0}^{\infty} N \cdot E_n}{\sum_{n=0}^{\infty} N}$$

$$= \frac{\sum_{n=0}^{\infty} N_0 e^{-E_n/KT} \cdot E_n}{\sum_{n=0}^{\infty} N_0 e^{-E_n/KT}}$$

$$\Rightarrow \boxed{\overline{E_n} = \frac{h\nu}{e^{h\nu/KT} - 1}} \quad \text{—— (II)}$$

Now energy density inside the cavity is :-

→ Jean's no.

$$U(\nu, T) = \frac{8\pi\nu^2}{c^3} \cdot \overline{E_n}$$

$$\Rightarrow \boxed{U(\nu, T) = \frac{8\pi\nu^2}{c^3} \cdot \frac{h\nu}{e^{h\nu/KT} - 1}}$$

which is Planck's Radiation Law.

One can write it in the term of wavelength

$$U(\lambda, T) = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda KT} - 1}$$

Now this law, when :-

- $\lambda \to 0$ becomes Wein's law
- $\lambda \to \infty$ becomes Rayleigh Jean's law.

Summary of Section 1

Wein's law : $I(\lambda, T) = \frac{a}{\lambda^5} \cdot e^{-b/KT}$

→ fails at longer wavelength

Rayleigh Jeans law : $U(\nu, T) = \frac{8\pi\nu^2}{c^3} \cdot KT$

→ fails at shorter wavelength & leads to UV catastrophe

~~Planck~~

Planck's Radiation law : $U(\nu, T) = \frac{8\pi\nu^2}{c^3} \cdot \frac{1}{e^{h\nu/KT} - 1}$

→ correctly fits the observed spectral curve.

at $\lambda \to 0$, becomes wein's law

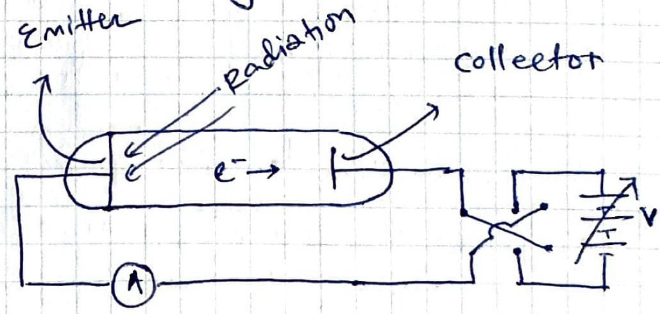
at $\lambda \to \infty$ " Rayleigh Jeans law.

② Photoelectric Effect

When electromagnetic wave of sufficiently high frequency hits the metal surface → electrons are ejected.

→ This effect ~~are~~ was first observed by Heinrich Hertz in 1887.

Observations made :-



- Min. frequency required to get e^- ejected. It is termed as Threshold Frequency.
- Kinetic Energy ~~has~~ of electrons ejected have a maximum value.
- Photoelectric (e^- ejection) emission is instantaneous process.
- Photoelectric current depends upon intensity of radiation and not frequency.

only last observation could be explained by classical Physics.

The first three observation could only be explained by considering Quantum Hypothesis. That is what Einstein did.

2.1 Einstein's Theory

Einstein continued using Planck's Quantum hypothesis. He explained incident radiation acts as stream of bundles having energy $h\nu$. When they collide with the electrons within the metal (provided that the incident stream have minimum frequency ν_0, known as threshold ~~freq~~ frequency). the energy is transferred and electrons are ejected.

→ Later on these bundles were called Photons.

And, corresponding to threshold frequency, the energy known as work function is necessary trade off. Hence maximum kinetic energy is :-

$$E_{max} = h\nu - W_0$$

Einstein's Photoelectric Equation.

W_0 → energy corresponding to threshold frequency, which is characteristic of metal.

Stopping Potential

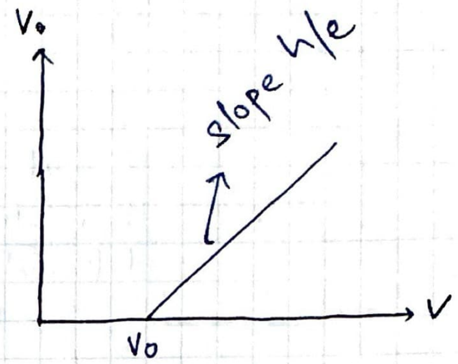
If the collector Plate is made negative, then photo-electrons will be repelled. And if the potential ~~such~~ applied is greater than corresponding energy, photoelectric flow would stop. That Potential is called Stopping Potential.

hence, Stopping potential = eV_0 = E_{max}

$$eV_0 = h\nu - W$$

or $$eV_0 = h\nu - h\nu_0$$

$$V_0 = \frac{h}{e}(\nu - \nu_0)$$



Thus the theory as explained established the correctness of Quantum Concept.

Using the slope h/e, Milikan found the value of h and thus Einstein won Nobel Prize in 1921 and Milikan won in 1923.

4. **Compton Effect**

↳ Collision of wave and Particle.

wave → photon

Particle → electron

if not Collision then interaction definitely.

observation

It was observed that – when X-rays is scattered using the atom having low atomic weight (eg. carbon)

↓

X-rays gets scattered

↓

COMPTON EFFECT { Two intensity peeks were observed. First peek at the incident wavelength & 2nd at wavelength slightly longer than the incident wavelength.

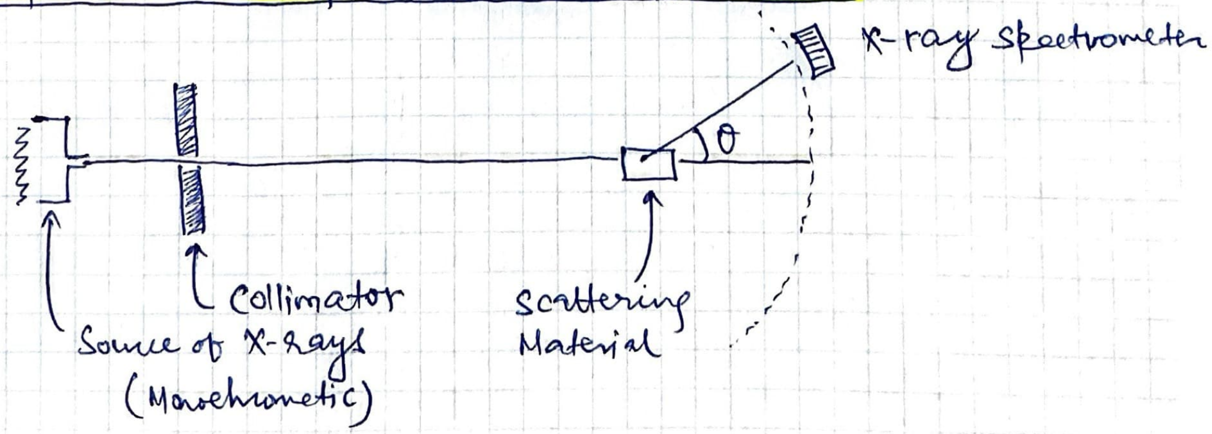
⇐ Provides most direct evidence of Particle Nature of Radiation

↑

it was a kind of Collision of Photon & electron.

Photon → Particle

Experimental Setup which led to observation



Classical Physics failed to explain the Compton Effect.

- As per classical Physics, an incoming em waves – drives the electron into S.H.M and the electron then re-radiates (scatters) the em wave (X-rays in our case) at the same frequency as the incident wave.
  ↳ or same wavelength

- Energy & Momentum are carried continously by the wave – that is – electron absorb and re-emit the X-rays with no net change in wavelength.

  → But in real, it was observed that scattered X-rays had slightly longer wavelength too.

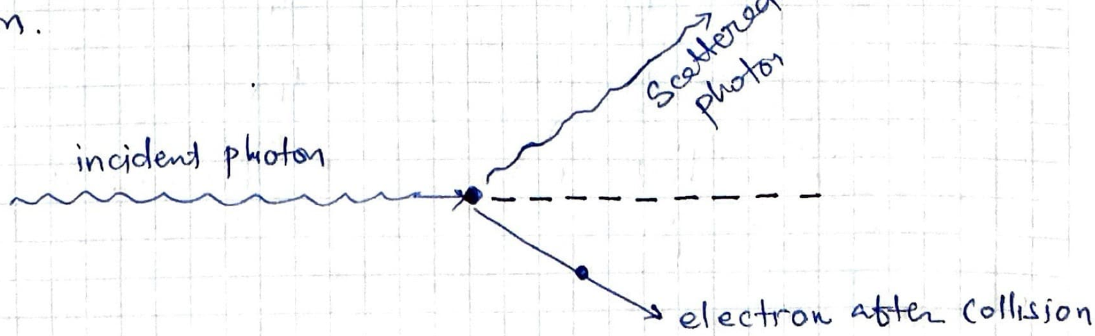
Using Discrete Energy Packet to Explain the Compton Effect

- Compton used Quantum theory → He assumed that X-rays are stream of photons with energy $h\nu$.

- And Photons from X-ray are making elastic collision with electron within the atom.

- Then he used – (i) Law of conservation of Energy
  (ii) " " " " Momentum

  to explain the wavelength shift of scattered Photon.



Initial conditions and final condition → energy & momentum of incident photon are $h\nu$ and $\frac{h\nu}{c}$

→ energy & momentum of scattered photon are $h\nu'$ and $\frac{h\nu'}{c}$

→ energy & momentum of electron at rest m_0c^2 and 0

→ energy & momentum of electron after collision $\sqrt{p^2c^2+(m_0c^2)^2}$ and p.

Applying law of conservation of energy :-

Initial energy of system = Final energy of the system

$$\Rightarrow \quad h\nu + m_0c^2 = h\nu' + \sqrt{p^2c^2 + m_0^2c^4} \quad \text{—— (I)}$$

Then applying law of conservation of Momentum :-

Initial Momentum of system in y direction (P_{iy}) = Final Momentum of system in x direction (P_{fy})

$$\Rightarrow \quad 0 + 0 = \frac{h\nu'}{c}\sin\theta + (-p\sin\phi) \quad \text{—— (II)}$$

Similarly,

~~P_{fx} = P_i~~

$$P_{ix} = P_{fx}$$

$$\Rightarrow \quad \frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\theta + p\cos\phi \quad \text{—— (III)}$$

Squaring and combining (II) and (III) we get:-

$$p^2\cos^2\phi + p^2\sin^2\phi = \left[\frac{h\nu}{c} - \frac{h\nu'}{c}\cos\theta\right]^2 + \left[\frac{h\nu'}{c}\sin\theta\right]^2$$

$$\Rightarrow \quad p^2 = \frac{h^2}{c^2}\left[(\nu - \nu'\cos\phi)^2 + \sin^2\theta\right]$$

$$\Rightarrow \quad \frac{p^2h^2}{c^2} = (\nu - \nu'\cos\phi)^2 + \sin^2\phi$$

Now we can get value of $\frac{p^2h^2}{c^2}$ from eq (I)

to get $\frac{2m_0c^2}{h}(\nu-\nu') = 2\nu\nu'(1-\cos\theta)$

$$\Rightarrow \quad \frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0c^2}(1-\cos\theta)$$

$$\Rightarrow \quad \lambda' - \lambda = \frac{h}{m_0c}(1-\cos\theta)$$

$$\Rightarrow \quad \Delta\lambda = \lambda_c(1-\cos\theta)$$

where λ_c is compton wavelength of electron which value **0.0242 Å**

Relation b/w θ and φ

from conservation of momentum we already have:-

$$\frac{pc}{h}\cos\phi = \nu - \nu'\cos\theta \quad \text{--- (IV)}$$

and

$$\frac{pc}{h}\sin\phi = \nu'\sin\theta \quad \text{--- (V)}$$

~~dividing (IV) by (V) we get --~~

~~$\cot\phi = \nu - \nu'\cos$~~

~~$\tan\phi = \frac{\nu'\sin\phi}{\nu - \nu'\sin}$~~

dividing eq(IV) by eq(V) we get -

$$\cot\phi = (1+\alpha)\tan\frac{\theta}{2}$$ where $\alpha = \frac{h\nu}{m_0c^2}$

Kinetic energy of Recoil Electron is :-

KE_e = Energy of incident photon − Energy of Scattered photon

$= h\nu - h\nu'$

$$= h\nu - h\left[\frac{\nu}{1+\alpha(1-\cos\theta)}\right]$$

$$KE_e = h\nu\,\frac{\alpha(1-\cos\theta)}{1+\alpha(1-\cos\theta)}$$

where we got $\frac{\nu}{\nu'} = 1+\alpha(1-\cos\theta)$ by using Momentum conservation equation.

~~and in term of ϕ one can write :-~~

and in term of ϕ one can write :-

$$E = h\nu\,\frac{2\alpha\cos^2\phi}{(1+\alpha)^2 - \alpha^2\cos^2\phi}$$